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Topographic Engineering Center

Handbook for Transformation of Datums, Projections, Grids, and Common Coordinate Systems

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ii

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EXECUTIVE SUMMARY

This document provides U.S. Army organizations and agencies with general guidance on selecting the appropriate methods for shifting between local geodetic datums and the World Geodetic System (WGS), and for converting Cartesian and map projection coordinates to and from geodetic coordinates. This guidance is provided to aid the Army community in selecting datum shift algorithms; developing, selecting, and maintaining software using these algorithms; and implementing this software to support operational units. Equations are furnished for map projections and datums commonly used within the Army. References are provided for other, less commonly encountered, map projections and datums.

Beneficial comments (corrections, recommendations, additions, deletions) or information that may be of use in improving this document should be addressed to the U.S. Army Engineer Research and Development Center's (ERDC), Topographic Engineering Center (TEC), Information Generation and Management Branch (IGMB) of the Topographic, Imagery, and Geospatial Research Division.

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PREFACE

This handbook was funded under the U.S. Army Topographic Engineering Center's (TEC) Digital Topographic Data Standards Program and co-authored at TEC by Messrs. James Ackeret, Frederick Gloeckler, Justin Simpson, Daniel Specht, and David Stoner.

The handbook is based on a draft of the Military Handbook, <u>Datums, Projections, Grids, and Common Coordinate Systems, Transformation of</u> (MIL-HDBK-600008), and was revised during the period July 1994 through July 1995. This handbook uses material from MIL-HDBK-600008 co-authored by Messrs. James Ackeret, Fred Esch, Chris Gard, Frederick Gloeckler, Daniel Oimoen, Juan Perez, Justin Simpson, Joe Watts, Tom Witte, and MAJ Harry Rossander.

This work was performed under the previous supervision of Messrs. Juan Perez, and Richard A. Herrmann and the supervision of John W. Hale, Chief, Standards Division and Regis J. Orsinger, Director, Digital Concepts and Analysis Center.

Mr. Robert Burkhart was the Director of ERDC's TEC at the time of re-publication of this handbook.

TEC is an element of the U.S. Army Engineer Research and Development Center (ERDC), U.S. Army Corps of Engineers. The Director of ERDC is Dr. James R. Houston and the Deputy to the Commander is A.J. Roberto, Jr.

CONTENTS

LIST OF FIGURES AND TABLES

Figures

Tables

HANDBOOK FOR TRANSFORMATION OF DATUMS, PROJECTIONS, GRIDS AND COMMON COORDINATE SYSTEMS

1   SCOPE

1.1 Scope. This handbook provides methods and parameter values to shift positions between many geodetic datums and provides methods for converting between geodetic coordinates, Cartesian coordinates, and map projection coordinates. Guidance is provided on selecting methods appropriate to the application, and on developing and testing software to implement these conversions.

1.2 Applicability. The methods provided in this handbook are used for a wide range of Army mapping, charting, and positioning applications. Equations are furnished for the map projections commonly used within the Army. References are provided for other map projections that occasionally may be encountered. Special high accuracy applications such as engineering, construction, and real estate boundary surveys are outside the scope of this handbook. Appendix B provides datum shift parameters for many datums that are currently available from the National Geospatial-Intelligence Agency (NGA); however, many more local datums exist. For guidance on datum shift applications or for datum shift parameters not covered by this handbook, contact TEC [please see Executive Summary].

1.3 Application guidance. Transformation methods and parameters should be appropriate to the application. A system analyst should do a thorough evaluation of the system requirements before developing detailed specifications. This evaluation should consider the intended use, data types, formats, and required accuracy of the output mapping, charting, and geodesy (MC&G) data, as well as the source and accuracy of the input data. Also, any hardware constraints must be considered. The methods presented in this handbook can then be examined for how well they can meet or be adapted to meet the requirements. Where more than one method will satisfy a basic requirement, give priority to Army-preferred methods for standardization purposes. Additional application guidance is furnished in Section 8.

2   REFERENCED DOCUMENTS

2.1 Government documents.

2.1.1 Specifications, standards, and handbooks. The following specifications, standards, and handbooks form a part of this document to the extent specified herein.

SPECIFICATIONS.   None

STANDARDS.

    MIL-STD-2401   Department of Defense World Geodetic System (WGS) (1994)

    STANAG 2211   Geodetic Datums, ellipsoids, grids, and grid references (Fifth Edition)(1991)

    HANDBOOKS.   None

2.1.2 Other Government documents, drawings, and publications. The following other Government documents, drawings, and publications form a part of this document to the extent specified herein. Unless otherwise specified, the issues are those cited in the solicitation. The date in parentheses indicates the edition used in preparation of this handbook.

Document ID/Agency	Title
DMA	Department of Defense Glossary of Mapping, Charting, and Geodetic Terms, Fourth Edition. (1981)
DMA TR 8350.2	Department of Defense World Geodetic System 1984, edition 1 (1993) http://164.214.2.59/GandG/pubs.html
DMA TR 8350.2-A	Supplement to Department of Defense World Geodetic System 1984 Handbook (1987) http://164.214.2.59/GandG/pubs.html
DMA TM 8358.1	Datums, Ellipsoids, Grids, and Grid Reference Systems, (1990)
DMA TM 8358.2	The Universal Grids: Universal Transverse Mercator (UTM) and Universal Polar Stereographic (UPS). (1989)
DMA Instruction 8000.1	Geodetic and Geophysical Sign Conventions and Fundamental Constants. (1988)
Geological Survey Professional Paper 1395	Map Projections - A Working Manual. (Snyder, 1987)
NGA TR 8350.2	Department of Defense World Geodetic System 1984, edition 3, Amendment 1 (2000) http://164.214.2.59/GandG/pubs.html

Document ID/Agency	Title
NOAA Manual NOS NGS 5	State Plane Coordinate System of 1983. (Stem, 1989)
NSWC/DL TR-3624	Map Projection Equations. (Pearson, 1977)
U.S. Department of Commerce, Coast and Geodetic Survey, Special Publication No. 251	Conformal Projections in Geodesy and Cartography. (Thomas, 1979)
Engineer Technical Letter No. 1110-1-147	Engineering and Design Conversion to North American Datum of 1983 (1990)
Federal Register v. 55, no. 155, Friday, August 10, 1990 Docket No. 900655-0165	Notice To Adopt Standard Method for Mathematical Horizontal Datum Transformation (NOAA, 1990)

Copies of DMA Instruction 8000.1 and GS Professional Paper 1395 are available from USGS Distribution Services, Box 25286, Denver, CO 80225 or from http://store.usgs.gov/.

Copies of DMA TM 8358.1, NGA TR 8350.2 and DMA TM 8358.2 are available from http://www.NGA.mil/GandG/pubs.html.

Copies of the following documents are only available to DoD personnel, who may obtain them from their supply person or NGA account holder: Glossary of Mapping, Charting, and Geodetic Terms; and DMA TR 8350.2-A.

Copies of the NOAA Manual NOS NGS 5 and the Coast and Geodetic Survey Special Publication No. 251 are available from National Geodetic Information Branch, N/NGS12, National Geodetic Survey, 1315 East-West Highway, Silver Spring, MD 20910-3282.

Copies of NSWC/DL TR 3624 (Accession Document Number = AD A037 381) are available from Defense Technical Information Center, 8725 John J. Kingman Rd., Ste. 0944, Fort Belvoir, VA 22060-6218 or from http://www.dtic.mil.

Copies of STANAG 2211 and MIL-STD 2401 are available from the Defense Printing Service Detachment Office, ATTN: DODSSP, Building 4/Section D, 700 Robbins Ave, Philadelphia, PA 19111.

2.2 Non-Government publications

Heiskanen, Weikko A., and Helmut Moritz; Physical Geodesy; Institute of Physical Geodesy, Technical University, Graz, Austria; 1967.

Krakiwsky, Edward J.; Conformal Map Projections in Geodesy; Department of Surveying Engineering, University of New Brunswick, Fredrickton, N.B., Canada; September 1973.

Rapp, Richard H.; Geometric Geodesy - Part I; Department of Geodetic Science and Surveying, The Ohio State University, Columbus, OH; 1984.

Rapp, Richard H.; Geometric Geodesy - Part II; Department of Geodetic Science and Surveying, The Ohio State University, Columbus, OH; 1987.

Toms, Ralph M; An Improved Algorithm for Geocentric to Geodetic Coordinate Conversion; Lawrence Livermore National Laboratory, University of California, Livermore, CA; 1996.

3 DEFINITIONS AND UNITS

3.1 Acronyms. The acronyms used in this handbook are defined as follows:

BIH	Bureau International de l'Heure
CONUS	continental United States
CTP	Conventional Terrestrial Pole
DMA	Defense Mapping Agency
DoD	Department of Defense
DTED	Digital Terrain Elevation Data
ECEF	earth-centered, earth-fixed
GPS	Global Positioning System
LGS	local geodetic system
MGRS	Military Grid Reference System
MC&G	mapping, charting and geodesy
MRE	multiple regression equation
MSL	mean sea level
NAD 27	North American Datum 1927
NAD 83	North American Datum 1983
NAVD 88	North American Vertical Datum of 1988
NGA	National Geospatial-Intelligence Agency
NGS	National Geodetic Survey
NGVD 29	National Geodetic Vertical Datum of 1929
NOAA	National Oceanic and Atmospheric Administration
NOS	National Ocean Survey
NSWC/DL	Naval Surface Weapons Center/Dahlgren Laboratory
TM	Transverse Mercator
UPS	Universal Polar Stereographic
UTM	Universal Transverse Mercator
WGS 72	World Geodetic System 1972
WGS 84	World Geodetic System 1984

3.2 Terms.

3.2.1 Convergence of the meridian (γ). The angle between the projection of the meridian at a point on a map and the projection North axis (typically represented by the y axis of the projection). The convergence of the meridian is positive in the clockwise direction.

3.2.2 Coordinate. Linear or angular quantities that designate the position that a point occupies in a given reference frame or system. Also used as a general term to designate the particular kind of reference frame or system, such as Cartesian coordinates or spherical coordinates.

3.2.3 <u>Datum</u>. Any numerical or geometrical quantity or set of such quantities specifying the reference coordinate system used for geodetic control in the calculation of coordinates of points on the earth. Datums may be either global or local in extent. A local datum defines a co-ordinate system that is used only over a region of limited extent. A global datum specifies the center of the reference ellipsoid to be located at the earth's center of mass and defines a coordinate system used for the entire earth.

3.2.3.1 <u>Horizontal datum</u>. A horizontal datum specifies the coordinate system in which latitude and longitude of points are located. The latitude and longitude of an initial point, the azimuth of a line from that point, and the semi-major axis and flattening of the ellipsoid that approximates the surface of the earth in the region of interest can define a horizontal datum.

3.2.3.2 <u>Vertical datum</u>. A vertical datum is the surface to which elevations are referred. A local vertical datum is a continuous surface, usually mean sea level, at which elevations are assumed to be zero throughout the area of interest.

3.2.4 <u>Earth-fixed</u>. Stationary with respect to the earth.

3.2.5 <u>Elevation</u>. Vertical distance measured along the local plumb line from a vertical datum, usually mean sea level or the geoid, to a point on the earth. Often used synonymously with orthometric height.

3.2.6 <u>Ellipsoid</u>. The surface generated by an ellipse rotating about one of its axes. Also called ellipsoid of revolution.

3.2.7 <u>Equator</u>. The line of zero geodetic latitude; the great circle described by the semi-major axis of the reference ellipsoid as it is rotated about the semi-minor axis.

3.2.8 <u>Equipotential surface</u>. A surface with the same potential, usually gravitational potential, at every point; a level surface.

3.2.9 <u>Geocentric Cartesian coordinates</u>. Cartesian coordinates (X, Y, Z) that define the position of a point with respect to the center of mass of the earth.

3.2.10 <u>Geodetic coordinates (geodetic position)</u>. The quantities of latitude, longitude, and geodetic height (ϕ, λ, h) that define the position of a point on the surface of the earth with respect to the reference ellipsoid. Also, imprecisely called geographic coordinates (see also geographic coordinates).

3.2.11 Geodetic height (ellipsoidal height, h). The height above the reference ellipsoid, measured along the ellipsoidal normal through the point in question. The geodetic height is positive if the point is outside the ellipsoid.

3.2.12 Geodetic latitude (ϕ). The angle between the plane of the Equator and the normal to the ellipsoid through the computation point. Geodetic latitude is positive north of the equator and negative south of the Equator (see Figure 4.2).

3.2.13 Geodetic longitude (λ). The angle between the plane of a meridian and the plane of the prime meridian. A longitude can be measured from the angle formed between the local and prime meridians at the pole of rotation of the reference ellipsoid, or by the arc along the Equator intercepted by these meridians. See Figure 4.2.

3.2.14 Geographic coordinates. The quantities of latitude and longitude that define the position of a point on the surface of the earth. See also geodetic coordinates.

3.2.15 Geoid. The equipotential surface of the earth's gravity field approximated by undisturbed mean sea level of the oceans. The direction of gravity passing through a given point on the geoid is perpendicular to this equipotential surface.

3.2.16 Geoid separation (N). The distance between the geoid and the mathematical reference ellipsoid as measured along the ellipsoidal normal. This distance is positive outside, or negative inside, the reference ellipsoid. Also called geoidal height; undulation of the geoid. See Figure 4.1.

3.2.17 Grid reference system. A plane-rectangular coordinate system usually based on, and mathematically adjusted to, a map projection in order that geodetic positions (latitudes and longitudes) may be readily transformed into plane coordinates and the computations relating to them may be made by the ordinary methods of plane surveying (see also 3.2.21).

3.2.18 Map projection. A function relating coordinates of points on a curved surface (usually an ellipsoid or sphere) to coordinates of points on a plane. A map projection may be established by analytical computation or, less commonly, may be constructed geometrically.

3.2.19 Map scale. The ratio between a distance on a map and the corresponding actual distance on the earth's surface.

3.2.20 Meridian. A north-south reference line, particularly a great circle through the geographical poles of the earth, from which longitudes and azimuths are determined; or the intersection of a plane forming a great circle that contains both geographic poles of the earth and

tion of a plane forming a great circle that contains both geographic poles of the earth and the ellipsoid.

3.2.21 Military Grid Reference System (MGRS). MGRS is a system developed by DMA for use with UTM and UPS projections.

3.2.22 Orthometric Height. See elevation.

3.2.23 Parallel. A line on the earth, or a representation thereof, that represents the same latitude at every point.

3.2.24 Prime (initial) meridian. A meridian from which the longitudes of all other meridians are reckoned. This meridian, of longitude 0°, was traditionally chosen to pass through the Greenwich Observatory in Greenwich, England. For new refined coordinate systems, the location of the prime meridian is defined by the International earth Rotation Service, Paris, France.

3.2.25 Reference ellipsoid. An ellipsoid whose dimensions closely approach the dimensions of the geoid; the exact dimensions are determined by various considerations of the section of the earth's surface concerned. Usually a bi-axial ellipsoid of revolution.

3.2.26 Scale factor (projection). A multiplier for reducing a distance in a map projection to the actual distance on the chosen reference ellipsoid.

3.3 Units. In this handbook all distances are expressed in meters. Angles are expressed in degrees, radians, or the combination of degrees, minutes, and seconds.

3.4 Sign conventions. The sign conventions used in this handbook are defined as follows:

   a. Elevation. Positive outside or above the vertical datum, negative inside or below the datum.

   b. Geodetic height. Positive outside the reference ellipsoid, negative inside the reference ellipsoid.

   c. Geodetic latitude. Positive in the northern hemisphere, negative in the southern hemisphere.

   d. Geodetic longitude. Zero at the prime meridian, positive 0° to 180°, eastward from the prime meridian, and negative 0° to 180°, westward from the prime meridian.

e. <u>Geoid separation</u>. Positive outside the reference ellipsoid, negative inside the reference ellipsoid.

f. <u>Cartesian Z</u>. Positive in the northern hemisphere.

3.5 <u>Unit conversion factors</u>. The standard unit of distance measurement is the meter; however, MC&G products in many parts of the world were produced using other units of measurement. The user must ascertain the units used in non-standard products and appropriately scale them for use in the equations given in this handbook. Some commonly used conversion factors are listed in Table 3.1.

TABLE 3.1. Conversion Factors.

Unit		Conversion
1 meter	=	3.28083333333 U.S. Survey feet
	=	3.28083989501 International feet
1 International foot	=	0.3048 meter (Exact)
1 U.S. Survey foot	=	$\frac{1200}{3937}$ meter (Exact)
	=	0.30480060960 meter
1 International Nautical mile	=	1,852 meters (Exact)
	=	6,076.10333333 U.S. Survey feet
	=	6,076.11548556 International feet
1 International Statute mile	=	1,609.344 meters (Exact)
	=	5,280 International feet (Exact)
1 degree	=	60 minutes
1 degree	=	3,600 seconds
1 minute	=	60 seconds
6,400 U.S. mils	=	360 degrees

3.5.1 <u>Degrees and radians</u>. A circle contains 360 degrees or 2π radians. If an angle contains d degrees or, equivalently, r radians, then d and r are related by

$$r = \frac{\pi \, d}{180} \qquad\qquad (3.1)$$

3.5.2 <u>Specifying the unit of angular measure</u>. In this handbook the unit of angular measure will be specified if it matters. A general rule of thumb is that angles must be in radians in an equation that was derived using calculus and trigonometry. Radians are the usual units in computer evaluation of trigonometric functions.

4   FUNDAMENTAL CONCEPTS

4.1 Introduction. An understanding of geodesy helps one understand how to represent the locations of points on or about the surface of the earth. Terms printed in boldface are defined in Section 3.2.

a.   The location of a point is described by a set of **coordinates**. Some types of coordinates, such as Cartesian (see Section 4.3.1), uniquely identify three-dimensional locations of points in space. Their use is computationally efficient in many applications; however, users often have difficulty visualizing the relationship of Cartesian coordinates to the surface of the earth. **Geodetic coordinates** (see Section 4.3.3) make visualization easier by providing horizontal coordinates on the surface of a figure approximating the shape of the earth and a vertical (height) coordinate above or below the reference surface.

b.   Geodetic coordinates are defined by a geodetic reference system. A geodetic reference system includes, but is not limited to, a reference surface (usually an ellipsoid) and a set of parameters positioning and orienting the reference surface with respect to the earth. Over the years, many geodetic reference systems have been developed for various localities. These systems are often called local datums and are not rigorously tied to each other.

c.   When applications require that two or more points be located in a common reference system, transformation between two coordinate systems may be needed. For example, both weapon and target must be located in the same reference system if the indirect fire computations are to be valid. As the ranges of weapons and target acquisition systems increased and positional data were derived from multiple sources, commanders often were confronted with the problem of using positions on more than one reference system. With the advent of satellite geodesy one can establish global geodetic reference systems and take measurements that allow local datums to be related to the global system. The standard global reference system for DoD is the World Geodetic System 1984 (WGS 84). The process of relating one reference system to another is called a datum shift, or datum transformation.

d.   Coordinates generally are represented alpha-numerically (e.g., latitude 38° 51' 22.45"N and longitude 77° 26' 34.56"W). Position data for an area often are represented graphically, as on a map. The planar representation on a map provides a distorted view of features on the curved earth's surface. Geodesists have developed various mathematical methods to minimize certain types of map distortions, depending on the intended use of the map. The process of relating coordinates of points on a curved surface (usually an ellipsoid or sphere) to coordinates of points on a plane is called **map projection**. The preferred map projections for DoD are UTM, used over most of the landmass, and Universal Polar Stereographic (UPS), used in the polar regions.

4.2 Reference surfaces. Reference surfaces are used as the basis for geodetic coordinate systems and elevations.

4.2.1 Reference ellipsoid. A reference surface called the **reference ellipsoid** (mathematically described in Section 4.3.2) is used for defining geodetic coordinates (Section 4.3.3) because it is regularly shaped, is mathematically tractable, and can approximate the surface of the earth. Since the earth is flattened at the poles, an oblate ellipsoid fits the shape of the earth better than does a sphere. An ellipsoid of revolution is the reference surface most commonly used in geodesy and is used throughout this handbook. A geodetic coordinate has a simply defined relationship with the reference ellipsoid and any other geodetic coordinate be-cause of the regularity and relative simplicity of the reference ellipsoid's shape. The parameters for the commonly used reference ellipsoids are listed in Appendix A. A reference ellipsoid is often chosen to fit the **geoid**.

4.2.2 Geoid. The **geoid** is a closed surface of constant gravity potential approximated by mean sea level (MSL) and the theoretical extension of MSL through land areas. For purposes of this handbook, the geoid is considered a reference surface from which elevations are measured. A reference ellipsoid may fit the geoid; for example, the maximum difference between the WGS 84 geoid model and the WGS 84 ellipsoid found in Appendix D, Table D.1, is 102 meters.

4.2.3 Relationships among topography, the geoid, and the reference ellipsoid. The relationships among topography, the geoid, and the reference ellipsoid are illustrated in Figure 4.1.

4.3 Earth-fixed coordinate systems. There are many coordinate systems used to represent positions on or near the earth. The coordinate systems used in this handbook are all **earth-fixed**. A coordinate system is earth-fixed if the axes are stationary with respect to the rotating earth. Two primary, earth-fixed coordinate systems, Cartesian and geodetic, are discussed in Sections 4.3.1 and 4.3.3, respectively. Conversions from one to the other are presented in Section 4.3.4.

4.3.1 Cartesian coordinates (X, Y, Z). The Cartesian coordinate system used in this handbook is a right-hand, rectangular, three-dimensional (X, Y, Z) coordinate system with an origin at (0, 0, 0). When the origin also is at the mass center of the earth, the coordinate system is said to be geocentric.
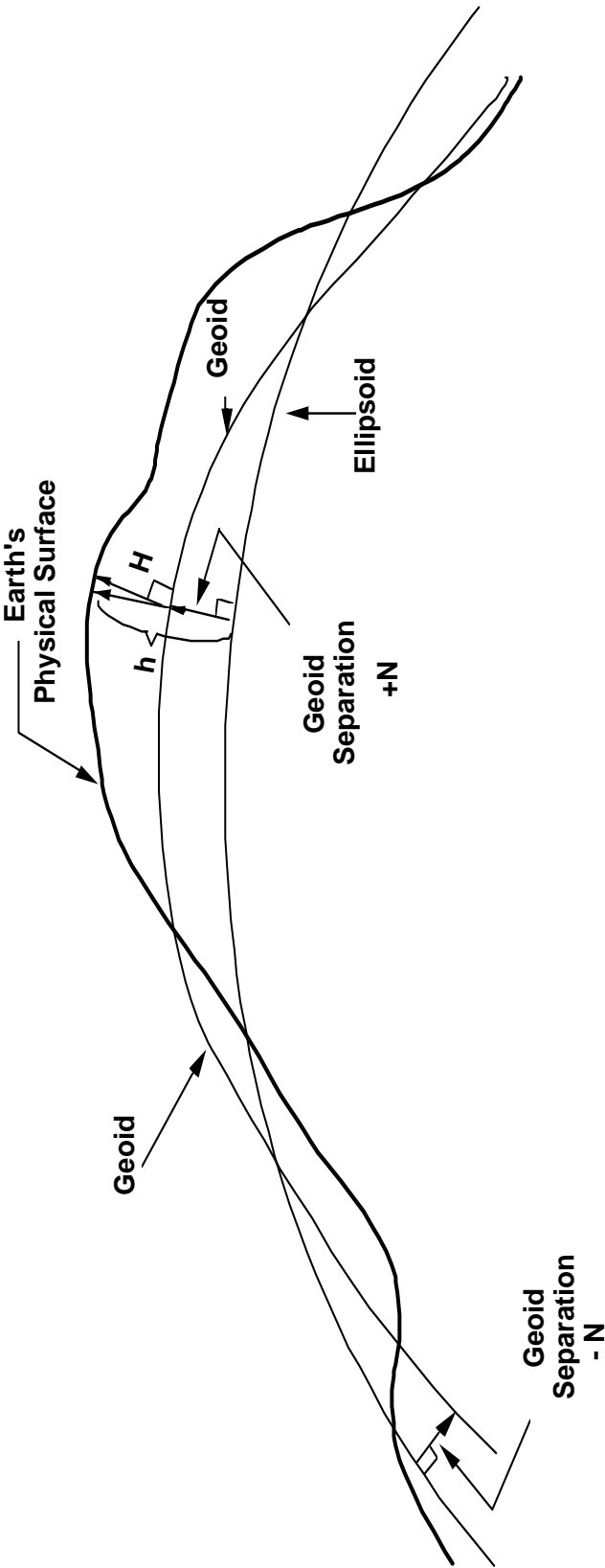
Figure 4.1. The relationship between the reference ellipsoid, the geoid, and the physical surface of the earth. Note the relationship of elevation (H), geodetic height (h), and geoid separation (N).

4.3.2 <u>Cartesian coordinate system/reference ellipsoid relationship</u>. The relationship of the Cartesian coordinate system to the reference ellipsoid is shown in Figure 4.2. The Z-axis, the axis of rotation (semi-minor axis) of the ellipsoid, is nearly parallel to the axis of rotation of the earth. The Z-coordinate is positive toward the North pole. The X-Y plane lies in the equatorial plane, the plane swept out by the semi-major axis as the ellipse is rotated. The X-axis lies along the intersection of the plane containing the prime (initial) meridian and the equatorial plane. The X-coordinate is positive toward the intersection of the prime meridian and equator. The reference ellipsoid satisfies the equation

$$\frac{X^2}{a^2} + \frac{Y^2}{a^2} + \frac{Z^2}{b^2} = 1 \tag{4.1}$$

Here, a and b, respectively, denote the semi-major and semi-minor axes of the ellipsoid. This surface can be generated by rotating an ellipse defined by

$$\frac{X^2}{a^2} + \frac{Z^2}{b^2} = 1 \tag{4.2}$$

about the Z-axis. Ideally, for global applications, the Z-axis coincides with the mean axis of rotation of the earth and the origin is the center of mass of the earth. Often, in practice, an ellipsoidal surface is fitted to a local area so that the origin and axes vary slightly from ideal coincidence.

4.3.2.1 <u>First eccentricity and flattening</u>. Two important constants are the (first) eccentricity ε and flattening f that are related to a and b by

$$\varepsilon^2 = \frac{a^2 - b^2}{a^2} \tag{4.3}$$

$$f = \frac{a - b}{a}$$

Frequently a and f are given, rather than a and b, as the defining parameters of a reference ellipsoid.
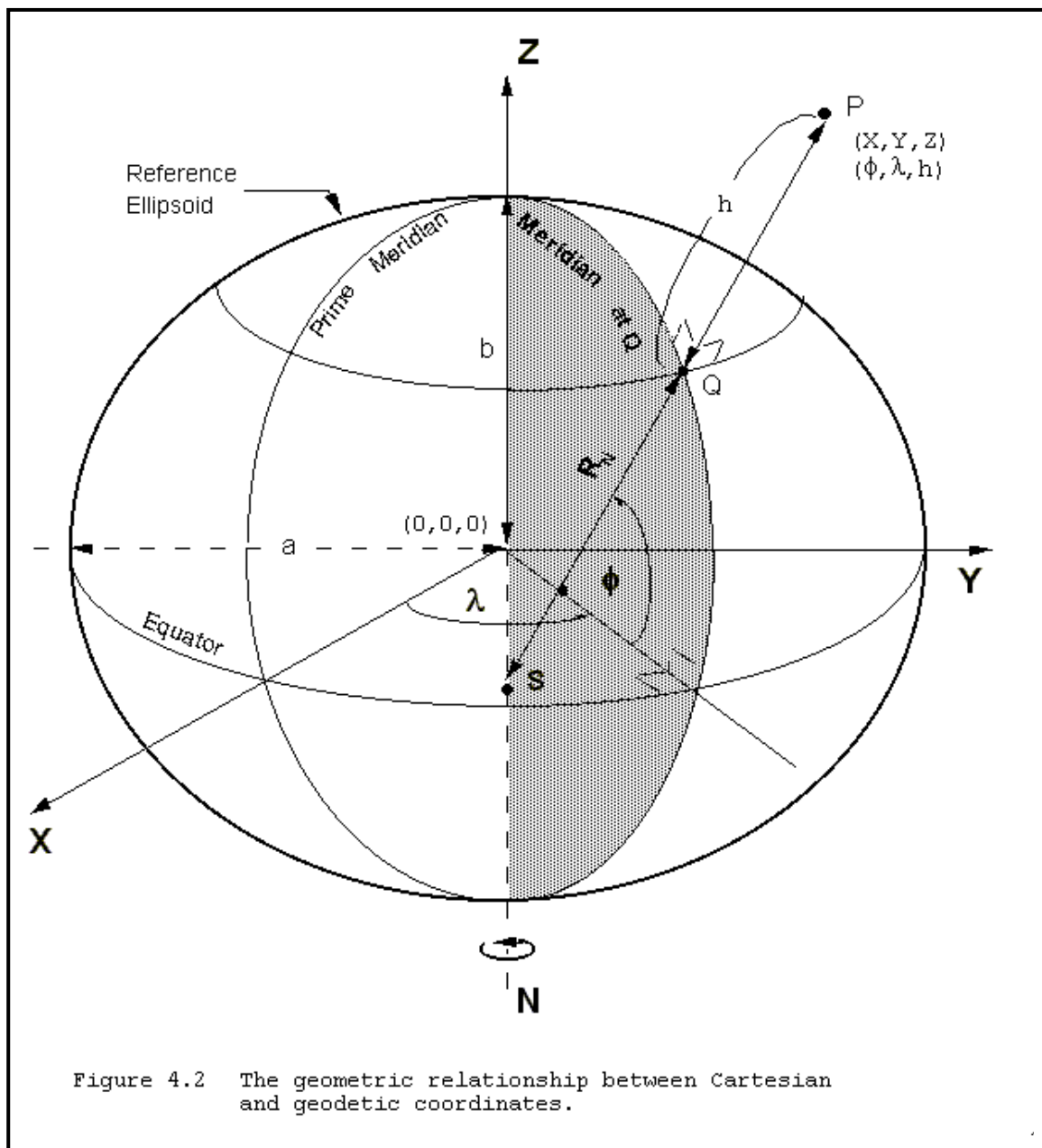
Figure 4.2. The geometric relationship between Cartesian and geodetic coordinates.

4.3.2.2 Ellipsoid parameters. Appendix A contains a list of reference ellipsoids and their parameters. These can be divided into two categories: those, such as the Bessel Ellipsoid, that are used to approximate the shape of the earth for a local area, and those, such as the WGS 84 Ellipsoid, that are used to yield a global approximation to the shape of the earth.

4.3.3 Geodetic coordinates (ϕ, λ, h). Geodetic coordinates consist of geodetic latitude (ϕ), geodetic longitude (λ), and geodetic height (h). Their relationship to the reference ellipsoid is shown in Figure 4.2. For an ellipsoid that satisfies equation 4.1, the normal SP intersects the ellipsoid at Q. The angle between the normal SP and the equatorial (X-Y) plane is called the **geodetic latitude** (ϕ) of point P. The meridian plane containing point P is defined as the half-plane containing the Z-axis and point P. The angle between the prime meridian (X-Z) plane and the meridian containing point P is the **geodetic longitude** (λ) of point P. Geodetic longitude is not defined when P lies on the Z-axis. The distance from Q to P is called the **geodetic height** (h).

4.3.3.1 Latitude and longitude limits.

    Latitude limits:
        $-90°$ to $+90°$
        $-90°$ at the South Pole
          $0°$ at the Equator
        $+90°$ at the North Pole

    Longitude limits:
        $0°$ at the Prime meridian
        $180°W$ ($-180°$) to $180°E$ ($+180°$)

4.3.4 Coordinate conversion. Two basic earth-fixed coordinate systems, geocentric Cartesian coordinates and geodetic coordinates, have been introduced in Sections 4.3.1 and 4.3.3. The following section explains how coordinates in one system can be transformed to the other. In Section 6, transformations between coordinates on two different reference ellipsoids are discussed. Map projections are treated in Section 7 and the general conversion process is summarized in Section 8.1.

4.3.4.1 Geodetic to Cartesian coordinate conversion. If geodetic coordinates (ϕ, λ, h) are known, then the Cartesian coordinates are given by

$$X = (R_N + h) \cos \phi \cos \lambda \tag{4.4}$$

$$Y = (R_N + h) \cos \phi \sin \lambda$$

$$Z = \left(\frac{b^2}{a^2} R_N + h \right) \sin \phi$$

where R_N, the radius of curvature in the prime vertical, is given by

$$R_N = \frac{a^2}{\sqrt{a^2 \cos^2\phi + b^2 \sin^2\phi}} = \frac{a}{\sqrt{1 - \varepsilon^2 \sin^2\phi}} \qquad (4.5)$$

See Figure 4.2. Equation sets 4.4 and 4.5 are exact. Calculating h from elevation (H) is discussed on section 6.5.

4.3.4.2 Cartesian to geodetic coordinate conversion.

4.3.4.2.1 Finding λ. In the reverse case, when (X, Y, Z) are known but (ϕ, λ, h) are unknown,

    for   the longitude (λ) is given by

$X \neq 0$, all Y,	$\lambda = \arctan\dfrac{X}{Y}$	(4.6)
$X = 0$, $Y > 0$,	$\lambda = 90°$ E ($+90°$)	
$X = 0$, $Y < 0$,	$\lambda = 90°$ W ($-90°$ or $270°$ E)	
$X = 0$, $Y = 0$,	λ is undefined	

Equation set 4.6 is exact. Take care to ensure that the evaluation of the arctan function yields a value of λ lying in the desired quadrant.

4.3.4.2.2 Finding ϕ. When (X, Y) is not (0, 0), the general equations for ϕ and h are more involved. The method presented for finding ϕ is an iterative procedure of Bowring. Following Rapp (1984, pp. 123-124), one finds an initial approximation to a variable β given by

$$\tan \beta_0 = \frac{a\,Z}{b\sqrt{X^2 + Y^2}} \qquad (4.7)$$

Once β_0 is obtained, substitute it for β in the equation

$$\tan \phi = \frac{Z + \varepsilon'^2\, b\, \sin^3 \beta}{\sqrt{X^2 + Y^2} - a\, \varepsilon^2 \cos^3 \beta} \qquad (4.8)$$

Here ε^2 and ε'^2 are given by

$$\varepsilon^2 = \frac{a^2 - b^2}{a^2} = 2f - f^{\,2} \qquad (4.9)$$

$$\varepsilon'^2 = \frac{a^2 - b^2}{b^2} = \frac{\varepsilon^2}{1 - \varepsilon^2}$$

This approximation to ϕ is substituted into

$$\tan \beta = (1 - f) \tan \phi \qquad\qquad (4.10)$$

to give an updated approximation to β. This procedure, of using the latest approximation of β (equation 4.10) to produce an updated approximation of ϕ (equation 4.8), and then using this new value of ϕ to update β, can be continued until the updated value of ϕ is sufficiently close to the previous value. For many purposes, one iteration is sufficient when using the Bowring method, so that ϕ can be found directly from equation 4.8, using the original value of β_0. Rapp (1984, p. 124) states that for terrestrial applications, one such iteration yields values accurate to within 0.1 millimeter. In the polar case $(X = 0, Y = 0, Z)$, the latitude (ϕ) is defined to be $+90°$ (90°N) for positive Z values and $-90°$ (90°S) for negative Z values.

Toms (1996) suggests an implementation of the Bowring method that reduces the number of trigonometric computations. For a general angle α, if

$$\tan \alpha = \frac{A}{B}$$

then

$$\sin \alpha = \frac{A}{\sqrt{A^2 + B^2}}$$

and

$$\cos \alpha = \frac{B}{\sqrt{A^2 + B^2}}$$

Consequently, it is not necessary to compute β in order to use equation 4.8. Compute the initial approximations

$$\sin \beta_0 = \frac{a\, Z}{\sqrt{a^2 Z^2 + b^2 (X^2 + Y^2)}}$$

$$\cos \beta_0 = \frac{b\sqrt{X^2 + Y^2}}{\sqrt{a^2 Z^2 + b^2 (X^2 + Y^2)}}$$

and substitute these into equation 4.8. If additional iterations are desired find $\tan \beta$ from equation 4.10. Then compute

$$\sin \beta = \frac{\tan \beta}{\sqrt{1 + \tan^2 \beta}}$$

$$\cos \beta = \frac{1}{\sqrt{1 + \tan^2 \beta}}$$

and substitute these into equation 4.8 to find an updated value of tan ϕ.

4.3.4.2.3 Calculating h. Once ϕ has been calculated, in non-polar areas h can be found from

$$h = \frac{\sqrt{X^2 + Y^2}}{\cos \phi} - R_N \qquad\qquad (4.11)$$

In polar regions it is preferable to use

$$h = \frac{Z}{\sin \phi} - R_N + \varepsilon^2 R_N \qquad\qquad (4.12)$$

where R_N is defined in equation 4.5.

NOTE: For many applications h is not required and need not be calculated. An exception to this statement is the computation of height or elevation for the Global Positioning System (GPS).

4.4 Representation of Geodetic Coordinates. Latitude is given before longitude. Latitude and longitude usually are numerically represented as: decimal degrees (e.g. 44.44° N); integer degrees and decimal minutes (e.g. 44° 44.44'S); or integer degrees, integer minutes and decimal seconds (e.g. 44° 44' 44.44" E). For clarity, numeric fields representing degrees, minutes and seconds should be ended by the appropriate symbol, respectively " ° ", " ' ", and " " ". Numeric fields with a value of zero should be indicated by a "00", and not left blank. The hemisphere of both latitude and longitude must be shown, preferably by placing an "N" or "S" at the end of numeric latitude and an "E" or "W" after numeric longitude.

4.5 Height Relationships.

4.5.1 Elevations. Throughout this handbook, the term **elevation** (H) is used to denote the distance of a point above the geoid or vertical reference surface as measured along the plumb line. A plumb line follows the direction of gravity and is perpendicular to all equipotential surfaces of the earth's gravity field that intersect it. Points lying outside the geoid are defined as having positive elevation. For the purposes of this handbook, elevation and **orthometric height** are considered equivalent. In practice, the reference surface for measuring elevations may not exactly coincide with the geoid.

4.5.2 Geoid separation. **Geoid separation** (N) is the distance from a reference ellipsoid to the geoid, measured along the ellipsoidal normal. This is equivalent to the geodetic height (see

Section 4.3.3) of a point on the geoid. Geoid separation is positive when the geoid lies outside the ellipsoid.

4.5.3 The relationships among H, h, and N. Geodetic height (h), geoid separation (N), and elevation (H) are depicted in Figure 4.1 and are related by equation 4.13.

$$h \approx H + N \tag{4.13}$$

4.5.3.1 Notational use of H and h. This handbook uses the classical notation of H for elevation and h for geodetic height.

5   GEODETIC SYSTEMS AND DATUMS

5.1 Introduction. A geodetic system serves as the framework for determining coordinates
of points with respect to the earth. Modern global geodetic systems, such as the World Geodetic
System 1984 (WGS 84), (see Section 5.4), have been established using techniques of satellite
geodesy. Typically, they are defined by a geocentric Cartesian coordinate system with the Z-axis
along the mean rotation axis of the earth and the X-axis through a longitude reference.

   a. DoD World Geodetic Systems have an associated reference ellipsoid and a geoid model.
       The geoid model is sometimes used as a reference surface for elevations. World Geodetic
       Systems provide a consistent framework for determining geodetic positions.

   b. Local geodetic systems are established by a variety of techniques. Typically, they consist
       of a geodetic datum (see Section 5.2) for horizontal control and a vertical datum (see Sec-
       tion 5.3) for elevations. Any given local geodetic system covers only a small fraction of
       the earth's surface. Local geodetic datums and vertical datums generally are independent
       of each other.

5.2 Geodetic (horizontal) datums. A geodetic datum has an earth-fixed reference ellipsoid
that may have been fit, in some manner, to the surface of the earth in the area of interest. Geo-
detic datums have parameters that define the size and shape of the ellipsoid.

5.2.1 Background. Historically, local geodetic datums have an origin, on the surface of
the earth, that relates the geodetic coordinate system to the ellipsoid. For these local datums, the
ellipsoid semi-minor axis generally does not coincide with the earth's mean rotation axis. How-
ever, the North American Datum 1983 (NAD 83) was developed using the same satellite geodesy
techniques used for global geodetic systems. This modern datum has an origin at the center of
the reference ellipsoid, and the semi-minor axis coincides with the mean rotation axis of the
earth. Without reference to the word vertical, the word datum, as used in this handbook, means
geodetic (horizontal) datum.

5.2.2 Geodetic datums and WGS 84. Over the years, hundreds of geodetic datums have
been developed for various locations. Currently, many local geodetic datums have been tied to
WGS 84. Coordinate transformations from local datums to WGS 84 are discussed in Section 6.

5.3 Vertical datums and elevations. By definition, a **vertical datum** is a surface of zero
elevation. Elevations are measured (positive upward) from the vertical datum. Ideally, a vertical
datum would closely approximate the geoid.

5.3.1 Background. As a practical matter, it is impossible to access the geoid surface directly for use as a vertical datum. Historically, tide gage measurements were averaged over many years to establish the local mean sea level (MSL) references for vertical datums. The National Geodetic Vertical Datum of 1929 (NGVD 29), which is based on tide gage measurements and precise leveling surveys, is estimated to be within a few meters of the geoid. There is greater uncertainty in the relationships between the geoid and other local vertical datums.

5.3.2 Vertical datums and mean sea level (MSL). The exact relationships between different vertical datums are unknown. Therefore, when shifting datums, all elevations are considered referenced to MSL regardless of the vertical datum used to establish them. For Army applications, two exceptions to this rule are:

a. Conversion of elevations derived from WGS 72 geodetic height measurements to WGS 84 coordinates (see Section 6.7.1).

b. Civil works, engineering and construction applications are outside the scope of this handbook.

5.3.3 Relationship between local vertical and horizontal datums. Generally, vertical datums and horizontal datums are independently defined. For example, NGVD 27 and NAVD 88 are the major vertical datums, while the North American Datum 1927 (NAD 27) and NAD 83 are the major horizontal datums used in North America.

5.4 World Geodetic System 1984 (WGS 84). A world geodetic system provides the basic reference frame, geometric figure, and gravimetric model for the earth, and provides the means for relating positions on various local geodetic systems to an earth-centered, earth-fixed (ECEF) coordinate system.

a. WGS 84 is the current standard DoD geodetic system. WGS 84 is the latest in a series of DoD-developed world geodetic systems. It is a replacement for WGS 72, the previous DoD standard ECEF world geodetic system.

b. The origin of the WGS 84 coordinate system is the center of mass of the earth. The WGS 84 Z-axis is parallel to the direction of the Conventional Terrestrial Pole (CTP), as defined by the Bureau International de l'Heure (BIH). The WGS 84 prime meridian is parallel to the BIH Zero meridian. The X-axis is the intersection of the plane of the WGS 84 reference meridian and the CTP equatorial plane. The epoch is 1984.0. The Y-axis completes a right-handed, earth-fixed Cartesian coordinate system.

c. The WGS 84 reference ellipsoid is a geocentric ellipsoid of revolution. The WGS 84 coordinate system origin and axes also serve as the geometric center and the X, Y, and Z axes of the WGS 84 ellipsoid. WGS 84 ellipsoid parameters are given in Appendix A.

d. The WGS 84 geoid model can be used to convert between elevations and geodetic heights (see Section 5.5).

e. Datum shift techniques and parameters needed to relate positions on many local datums to WGS 84 also were developed as part of WGS 84 (see Section 6 and Appendices B and E).

f. Only those portions of WGS 84 related to the purpose of this handbook were described above. A comprehensive view of WGS 84 can be obtained from DMA TRs 8350.2 (1993), 8350.2-A (1987), and 8350.2-B (1987).

5.5 WGS 84 referenced elevations. Sometimes elevations must be generated from geodetic heights, without reference to a local vertical datum. For example, satellite positioning systems, like the GPS, establish coordinates geometrically and cannot directly measure elevations. An elevation can be obtained from a WGS 84 geodetic height using:

$$H \approx h_{WGS\ 84} - N_{WGS\ 84} \qquad\qquad (5.1)$$

where $N_{WGS\ 84}$ is the value of WGS 84 geoid separation for the WGS 84 geodetic position of the point. NIMA, in collaboration with NASA and the Ohio State University, developed an earth gravity model, Earth Gravitational Model 1996 (EGM96). For DoD applications this model replaces the original WGS 84 gravitational model. EGM96 uses a spherical harmonic expansion of the gravitational potential, and includes 130,676 coefficients. Software, the gravity model coefficients, and the 15 minute grid of geoid separation values, accurate to 1 meter or better, is available at http://cddisa.gsfc.nasa.gov/926/egm96/egm96.html.

6   DATUM SHIFTS

6.1 Introduction. Several methods are available to shift coordinates from one geodetic
datum to another. To develop these shifts, coordinates of one or more physical locations must be
known on both datums. Typically, for the local datum to WGS 84 shifts presented in this hand-
book, the WGS 84 coordinates of local datum survey control points were determined using Dop-
pler satellite observations. Two classes of datum shift methods are geometric transformation and
polynomial fitting techniques. Only geometric transformations are discussed here.

    a. The generalized geometric transformation model assumes the origins of the two coordi-
        nate systems are offset from each other; the axes are not parallel; and there is a scale dif-
        ference between the two datums. This **seven-parameter model** is discussed in Section
        6.2. Data from at least three well-spaced positions are needed to derive a seven-
        parameter geometric transformation.

    b. The number of parameters in a geometric transformation model can be reduced. The
        commonly used **three-parameter model** neglects rotations between coordinate systems
        axes and scale differences between datums. Only the origins are offset. The three-
        parameter model is discussed in Section 6.3. Data from one or more positions are re-
        quired to derive a three-parameter geometric transformation.

    c. The WGS 72 to WGS 84 transformation, given in Section 6.4, is a special five-parameter
        geometric transformation. When WGS 84 superseded WGS 72 as the DoD-preferred ref-
        erence system, this transformation was created specifically to transform WGS 72 coordi-
        nates to WGS 84 coordinates.

    d. Heights used for datum shifting are treated in Section 6.5. Shifting of vertical datums is
        discussed in Section 6.7.

6.1.1 Shifting between two local datums. The datum shift methods given in this handbook
are for shifting from a local datum to WGS 84 and from WGS 84 to a local datum. When shifts
are needed between two local datums, shift the coordinates on the first local datum to WGS 84,
then shift the WGS 84 coordinates to the second local datum.

6.1.1.1 Shifting from NAD 27 to NAD 83. The software package NADCON has been recommended as the standard method for transformations between the North American Datum of 1927 (NAD27) and the North American Datum of 1983 (NAD 83) by The Federal Geodetic Control Committee (Federal Register.) CORPSCON is a program that incorporates NADCON and the NGS program VERTCON, which performs vertical conversions between NGVD 29 and NAVD 88. CORPSCON may be found under "Software Available" at http://www.tec.army.mil. Because NADCON, VERTCON and CORPSCON are survey tools, they are beyond the scope of the handbook.

6.2 Seven-parameter geometric transformation. The seven-parameter transformation, the most general transformation between local and global Cartesian coordinates, is considered in DMA TR 8350.2-A (1987). The underlying assumption for this seven-parameter model is that the local and WGS 84 Cartesian axes differ by seven parameters: three rotational parameters (ω, ε, ψ), a scale change ΔS, and three origin shift parameters (ΔX, ΔY, ΔZ). See Figure 6.1.
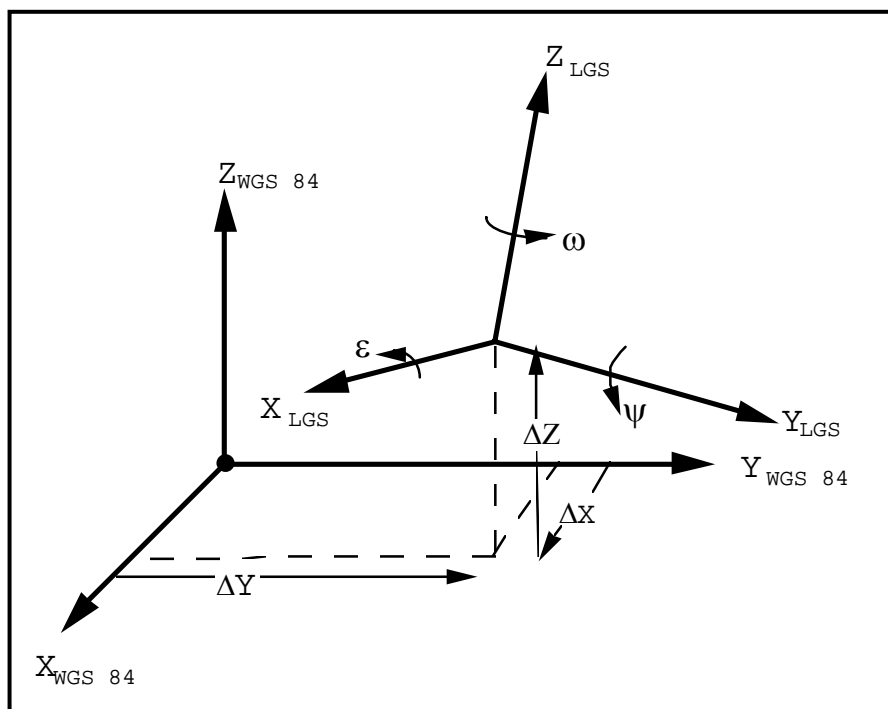
Figure 6.1. Relationship between coordinate axes in a seven-parameter geometric transformation.

6.2.1 Transformation to WGS 84 Cartesian coordinates. For the seven-parameter model, the local and WGS 84 Cartesian coordinates are related by

$$X_{WGS\ 84} = X_{LGS} + \Delta X + \omega\, Y_{LGS} - \psi\, Z_{LGS} + \Delta S\, X_{LGS} \qquad (6.1)$$

$$Y_{WGS\ 84} = Y_{LGS} + \Delta Y - \omega\, X_{LGS} + \varepsilon\, Z_{LGS} + \Delta S\, Y_{LGS}$$

$$Z_{WGS\ 84} = Z_{LGS} + \Delta Z + \psi\ X_{LGS} - \varepsilon\ Y_{LGS} + \Delta S\ Z_{LGS}$$

The subscript LGS designates Local Geodetic System coordinates and the subscript WGS 84 designates World Geodetic System 1984 coordinates.

6.2.2 <u>Transformation to LGS Cartesian coordinates</u>. The equations for converting WGS 84 geocentric Cartesian coordinates to local Cartesian coordinates are

$$X_{LGS} = X_{WGS\ 84} - \Delta X - \omega\ Y_{WGS\ 84} + \psi\ Z_{WGS\ 84} - \Delta S\ X_{WGS\ 84} \qquad (6.2)$$

$$Y_{LGS} = Y_{WGS\ 84} - \Delta Y + \omega X_{WGS\ 84} - \varepsilon\ Z_{WGS\ 84} - \Delta S\ Y_{WGS\ 84}$$

$$Z_{LGS} = Z_{WGS\ 84} - \Delta Z - \psi\ X_{WGS\ 84} + \varepsilon\ Y_{WGS\ 84} - \Delta S\ Z_{WGS\ 84}$$

6.2.3 <u>Parameter values</u>. The parameters ΔX, ΔY, ΔZ, ω, ε, ψ, and ΔS, for the European 1950 and Ordinance Survey of Great Britain 1936 datums are tabulated in Appendix E. Use of seven-parameter datum shifts for some applications in Europe and Great Britain are prescribed by STANAG 2211. Since the values of the rotational parameters ω, ε, and ψ are given in the table in arc seconds, these values must be converted to radians before using equation sets 6.1 and 6.2.

6.2.4 <u>Accuracy of seven-parameter transformation</u>. The RMS differences between WGS 84 surveyed and transformed coordinates using the seven-parameter datum transformations are given in Appendix E. For some datums, the accuracies for the seven-parameter transformation are not an improvement over those for the three-parameter transformation. Equation sets 6.1 and 6.2 provide good approximations to a rigorous orthogonal transformation when ω, ε, ψ, and Δs are small.

6.3 <u>Three-parameter(ΔX, ΔY, ΔZ) geometric transformation</u>. Frequently, it is not practical to determine the parameters for a seven-parameter geometric datum shift or implement such a shift method. Often, only an origin shift, three-parameter model is used as shown in Figure 6.2. The three-parameter datum transformation satisfies the requirements of most mapping and charting applications. The three-parameter model can be applied directly to Cartesian coordinates (see Sections 6.3.1 and 6.3.2) or used to compute shifts in geodetic coordinates via the Molodensky equations (see Section 6.3.3)

   a. The datum shift parameters (ΔX, ΔY, ΔZ) are the coordinates of the origin of the local reference ellipsoid in the WGS 84 Cartesian coordinate system.

   b. Datum shift parameters for many local geodetic systems are listed in Appendix B, Table B.1. The area covered by the datum shift parameters, the local datum ellipsoid, and the

estimates of the errors in the datum shift parameters also are given. Derivation of ΔX, ΔY, and ΔZ datum shift parameters is discussed in DMA TR 8350.2-A (1987).
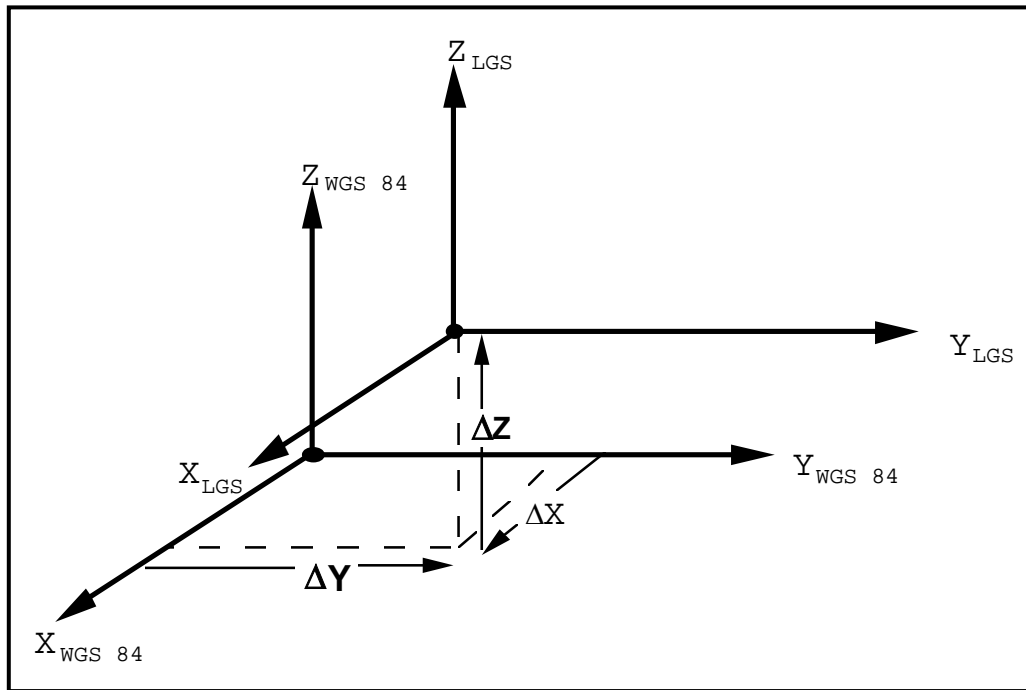

Figure 6.2. Relationship between coordinate axes in a three-parameter geometric transformation.

6.3.1 Transformation to WGS 84 Cartesian coordinates. In certain cases, ΔX, ΔY, and ΔZ datum shift parameters can be applied directly to convert local geodetic system coordinates to WGS 84 Cartesian coordinates as follows:

$$X_{WGS\ 84} = X_{LGS} + \Delta X \qquad\qquad\qquad (6.3)$$

$$Y_{WGS\ 84} = Y_{LGS} + \Delta Y$$

$$Z_{WGS\ 84} = Z_{LGS} + \Delta Z$$

Equation set 6.3 is a subset of equation set 6.1 with ω, ψ, ε, and ΔS set to zero.

6.3.1.1 Three-step method: Transformation to WGS 84 geodetic coordinates. Local geodetic coordinates can be shifted to WGS 84 coordinates in three steps:

a. Convert local geodetic coordinates to local geocentric Cartesian coordinates using equation set 4.4. Ensure that local ellipsoid parameters are used.

b. Shift the local geocentric Cartesian coordinates to WGS 84 geocentric Cartesian coordinates using equation set 6.3.

c. Convert WGS 84 geocentric Cartesian coordinates to WGS 84 geodetic coordinates using the methods discussed in Section 4.3.4.2. Ensure that WGS 1984 ellipsoid parameters are used.

6.3.2 <u>Transformation to local geodetic coordinates</u>. To convert WGS 84 Cartesian coordinates to the local geodetic system, apply ΔX, ΔY, and ΔZ datum shifts as follows

$$X_{LGS} = X_{WGS\ 84} - \Delta X \qquad\qquad (6.4)$$

$$Y_{LGS} = Y_{WGS\ 84} - \Delta Y$$

$$Z_{LGS} = Z_{WGS\ 84} - \Delta Z$$

Equation set 6.4 is a subset of equation set 6.2 with ω, ψ, ε, and ΔS set to zero.

6.3.2.1 <u>Three-step method: Transformation to local geodetic coordinates</u>. To shift WGS 84 geodetic coordinates to local geodetic coordinates

a. Convert WGS 84 geodetic coordinates to WGS 84 geocentric Cartesian coordinates using equation set 4.4. Ensure that WGS 84 ellipsoid parameters are used.

b. Shift the WGS 84 geocentric Cartesian coordinates to local geocentric Cartesian coordinates using equation set 6.4.

c. Convert the local geocentric Cartesian coordinates to local geodetic coordinates using a method of Section 4.3.4.2. Ensure that local ellipsoid parameters are used.

6.3.3 <u>Molodensky shifts</u>. The standard Molodensky method is an approximation to the three-step transformation methods of Sections 6.3.1.1 and 6.3.2.1. To use the Molodensky methods to transform local geodetic coordinates to WGS 84 geodetic coordinates, calculate $\Delta\phi$, $\Delta\lambda$, and Δh shifts using the standard Molodensky formulas (Rapp, 1987).

$$\Delta\phi = [-\Delta X \sin\phi \cos\lambda - \Delta Y \sin\phi \sin\lambda + \Delta Z \cos\phi + \frac{\varepsilon^2 \sin\phi \cos\phi}{W} \Delta a$$

$$+ \sin\phi\cos\phi(2N + \varepsilon'^2 M \sin^2\phi)(1-f)\Delta f]/[M + h] \qquad (6.5)$$

$$\Delta\lambda = \frac{[-\Delta X \sin\lambda + \Delta Y \cos\lambda]}{[(N + h)\cos\phi]}$$

$$\Delta h = \Delta X \cos\phi \cos\lambda + \Delta Y \cos\phi \sin\lambda + \Delta Z \sin\phi - W \Delta a + \frac{a(1-f)}{W} \Delta f \sin^2\phi$$

in which

$$W^2 = 1 - \varepsilon^2 \sin^2 \phi$$

$$M = \frac{a(1 - \varepsilon^2)}{W^3}$$

$$N = \frac{a}{W}$$

$$\phi = \phi_{LGS}, \lambda = \lambda_{LGS}$$

$$a = a_{LGS}, b = b_{LGS}, f = f_{LGS}$$

$$\varepsilon^2 = \frac{a^2 - b^2}{a^2} = 2f - f^2$$

$$\varepsilon'^2 = \frac{a^2 - b^2}{b^2} = \frac{\varepsilon^2}{1 - \varepsilon^2}$$

The units of $\Delta\phi$ and $\Delta\lambda$ are radians. Note that $N = R_N$ of equation set 4.5.

Note: Most applications do not require shifting geodetic heights, so Δh does not need to be computed or applied. Geodetic heights, which are used to compute datum shift parameters, are a special construct that may not be well related to the local geodetic system. Do not use Δh to shift local geodetic heights. If this is a requirement, seek competent geodetic counsel.

a. Apply the $\Delta\phi$, $\Delta\lambda$, and Δh shifts as follows:

$$\phi_{WGS\ 84} = \phi_{LGS} + \Delta\phi_{LGS} \qquad\qquad (6.6)$$

$$\lambda_{WGS\ 84} = \lambda_{LGS} + \Delta\lambda_{LGS}$$

$$h_{WGS\ 84} = h_{LGS} + \Delta h_{LGS}$$

where: $\Delta\phi_{LGS}$, $\Delta\lambda_{LGS}$, Δh_{LGS} are computed from equation set 6.5 using ϕ_{LGS}, λ_{LGS}, h_{LGS}, semi-major axis and flattening for the local geodetic system. ΔX, ΔY, and ΔZ are found in Appendix B. Δa and Δf are calculated as follows:

$$\Delta a = a_{WGS\ 84} - a_{LGS}$$

$$\Delta f = f_{WGS\ 84} - f_{LGS}$$

Note: See section 6.5 for use of h_{LGS}.

b. The Molodensky formulas also can be used to convert from WGS 84 to the local geodetic system

$$\phi_{\text{LGS}} = \phi_{\text{WGS 84}} + \Delta\phi_{\text{WGS 84}} \qquad\qquad (6.7)$$

$$\lambda_{\text{LGS}} = \lambda_{\text{WGS 84}} + \Delta\lambda_{\text{WGS 84}}$$

$$h_{\text{LGS}} = h_{\text{WGS 84}} + \Delta h_{\text{WGS 84}}$$

where $\Delta\phi_{\text{WGS 84}}$, $\Delta\lambda_{\text{WGS 84}}$, and $\Delta h_{\text{WGS 84}}$ are computed using $\phi_{\text{WGS 84}}$, $\lambda_{\text{WGS 84}}$, $h_{\text{WGS 84}}$, WGS 84 semi-major axis and flattening, and the signs of ΔX, ΔY, and ΔZ, as given in Appendix B, are reversed. Δa and Δf are calculated as follows

$$\Delta a = a_{\text{LGS}} - a_{\text{WGS 84}}$$

$$\Delta f = f_{\text{LGS}} - f_{\text{WGS 84}}$$

Note that ellipsoid parameters a and f are found in Table A.1, Appendix A.

6.3.4 $\underline{\Delta X, \Delta Y, \text{and } \Delta Z \text{ shift values}}$. Appendix B (Table B.1) contains the ΔX, ΔY, and ΔZ values for the three-parameter methods of converting local geodetic systems to WGS 84. The accuracies, in terms of error estimates for ΔX, ΔY, and ΔZ, are given in Appendix B. A further discussion of the accuracy of these parameter shifts will be given in Section 6.3.5.

6.3.5 $\underline{\text{Accuracy of } \Delta X, \Delta Y, \text{and } \Delta Z \text{ shift parameters}}$. Error estimates for ΔX, ΔY, and ΔZ are tabulated in Appendix B. These estimates include the errors associated with the Doppler stations as well as the residual differences between transformed local system coordinates and the reference WGS 84 coordinates used to develop the datum shifts. (A Doppler station refers to a position where the WGS 84 coordinates were obtained by observing and processing of TRANSIT satellite data.)

a. For some datums, datum shifts were developed using a single Doppler station. For these datums, no measure of internal consistency of the datum shifts is calculable, and NGA has assigned an accuracy value of 25 meters in each coordinate. These error statistics do not reflect any errors in the coordinates used to compute the datum shift.

b. Datum shifts for five datums were calculated from sources other than Doppler stations for WGS 84 geodetic coordinates. No error statistics are provided for these datums.

c. Geodetic coordinate difference error estimates can be approximated from rectangular coordinate errors using the following spherical relationships:

$$\sigma_{\Delta\phi} = \sqrt{(\sigma_{\Delta X} \sin\phi \cos\lambda)^2 + (\sigma_{\Delta Y} \sin\phi \sin\lambda)^2 + (\sigma_{\Delta Z} \cos\phi)^2} \qquad (6.8)$$

$$\sigma_{\Delta\lambda} = \sqrt{(\sigma_{\Delta X} \sin\lambda)^2 + (\sigma_{\Delta Y} \cos\lambda)^2}$$

$$\sigma_{\Delta h} = \sqrt{(\sigma_{\Delta X} \cos\phi \cos\lambda)^2 + (\sigma_{\Delta Y} \cos\phi \sin\lambda)^2 + (\sigma_{\Delta Z} \sin\phi)^2}$$

Note that these relationships neglect any correlations that may exist between the error estimates for ΔX, ΔY and ΔZ.

6.3.6 Local datum distortion. For some local datums, such as the Australian Geodetic datums, one set of ΔX, ΔY, and ΔZ datum shift parameters provides a consistent measure of the datum shifts throughout the datum. For other datums, ΔX, ΔY, and ΔZ shift parameters for the whole local datum, called mean-value datum shift parameters, poorly fit one or more regions within the datum. Regional variation in the datum shift parameters may result from using different, poorly connected surveys to establish the geodetic control on the local datum; from rotation of the local datum; and from scale differences. An extreme example of regional datum shift variation is the ΔY component for the ARC 1950 datum. The mean-value ΔY is –90 meters; however, ΔY ranges from –5 to –108 meters for different countries covered by the ARC 1950 datum. Appendix B (Table B.1) contains regional ΔX, ΔY, and ΔZ datum shift parameters for some local datums. For these datums, one should base the selection of mean or regional ΔX, ΔY, and ΔZ shift values on the accuracy required for the application. Only mean value three-parameter shifts are available for the remaining datums listed in Table B.1.

6.4 WGS 72 to WGS 84 transformation. WGS 84 has replaced WGS 72 as the accepted geodetic reference system for most DoD applications. The WGS 84 system, developed through a more extensive set of satellite-derived surface data than was available at the start of WGS 72, is an improved geometric and gravitational model of the earth. Whenever possible, WGS 72 coordinates should be converted to the WGS 84 reference system.

6.4.1 Selecting a conversion method. Before selecting a WGS 72 to WGS 84 conversion method, find the source of the WGS 72 coordinates. Select the conversion method using the guidance in Table 6.1.

Table 6.1. Selecting a WGS 72 to WGS 84 transformation method.

	Source of WGS 72 Coordinates	**Transformation Method**
1.	Doppler satellite station	Direct (6.4.2)
2.	Local datum to WGS 72 transformation. Local datum coordinates known.	Local datum to WGS 84 transformation. (6.2, or 6.3)
3.	Local datum to WGS 72 transformation. Local datum coordinates unknown.	Two-Step (6.4.3) Shift method known.
4.	Local datum to WGS 72 transformation using localized datum shift parameters derived for nearby WGS 72 Doppler station. Local datum coordinates unknown.	Direct (6.4.2)
5.	Unknown	Direct (6.4.2) Accuracy unknown.

6.4.2 <u>Direct WGS 72 to WGS 84 transformation</u>. The direct WGS 72 to WGS 84 transformation reflects fundamental changes between the WGS 72 and WGS 84 systems. Changes include a shift in coordinate system origin, a shift in longitudinal reference (initial meridian), a change in system scale, and changes in ellipsoidal parameters. For WGS 72 coordinates not originating in the WGS 72 system (i.e., non-Doppler-derived coordinates), take care when employing the direct WGS 72 to WGS 84 transformation, as any inaccuracies and uncertainties inherent in the WGS 72 coordinates will be directly transferred into the derived WGS 84 coordinates. These inaccuracies and uncertainties are generally a result of local geodetic system to WGS 72 coordinate transformations using mean rather than localized datum shifts. Thus, every effort should be made to find the source of all WGS 72 coordinates before doing a direct WGS 72 to WGS 84 transformation.

6.4.2.1 <u>Conversion equations</u>. It is important to note that the relationship between the WGS 72 coordinates and the WGS 84 coordinates cannot be described by a three-parameter model (see Section 6.3).

a.   The equations for the direct WGS 72 to WGS 84 geodetic coordinate transformation are

$$\phi_{WGS\ 84} = \phi_{WGS\ 72} + \Delta\phi \qquad\qquad\qquad (6.9)$$

$$\lambda_{WGS\ 84} = \lambda_{WGS\ 72} + \Delta\lambda$$

$$h_{WGS\ 84} = h_{WGS\ 72} + \Delta h$$

where

$$\Delta\phi = \frac{4.5\ \cos\phi_{WGS\ 72}}{a_{WGS\ 72}\ Q} + \frac{\Delta f\ \sin 2\phi_{\ WGS\ 72}}{Q}\ \text{(arc seconds)}$$

$$\Delta\lambda = 0.554\ \text{(arc seconds)}$$

$$\Delta h = 4.5 \ \sin\phi_{WGS\ 72} + a_{WGS\ 72} \ \Delta f \ \sin^2 \phi_{WGS\ 72} - \Delta a + \Delta r \quad \text{(meters)}$$

$$\Delta f = 0.3121057 \ \text{x} \ 10^{-7}$$

$$a_{WGS\ 72} = 6378135 \quad \text{(meters)}$$

$$\Delta a = 2.0 \quad \text{(meters)}$$

$$\Delta r = 1.4 \quad \text{(meters)}$$

$$Q = \frac{\pi}{180 \cdot 3600}$$

b. The equations for the direct WGS 84 to WGS 72 geodetic coordinate transformation are:

$$\phi_{WGS\ 72} = \phi_{WGS\ 84} + \Delta\phi \qquad\qquad (6.10)$$

$$\lambda_{WGS\ 72} = \lambda_{WGS\ 84} + \Delta\lambda$$

$$h_{WGS\ 72} = h_{WGS\ 84} + \Delta h$$

where

$$\Delta\phi = \frac{-4.5 \ \cos\phi_{WGS\ 84}}{a_{WGS\ 84} \ Q} - \frac{\Delta f \ \sin 2\phi_{WGS\ 84}}{Q} \quad \text{(arc seconds)}$$

$$\Delta\lambda = -0.554 \quad \text{(arc seconds)}$$

$$\Delta h = -4.5 \ \sin\phi - a_{WGS\ 84} \ \Delta f \ \sin^2 \phi_{WGS\ 84} + \Delta a - \Delta r \ \text{(meters)}$$

$$\Delta f = 0.3121057 \ \text{x} \ 10^{-7}$$

$$a_{WGS\ 84} = 6378137 \quad \text{(meters)}$$

$$\Delta a = 2.0 \quad \text{(meters)}$$

$$\Delta r = 1.4 \quad \text{(meters)}$$

$$Q = \frac{\pi}{180 \cdot 3600}$$

Elevation based on the WGS 84 geoid model will be different from elevations based on the WGS 72 geoid model due to differences in the geoid models (see Section 6.7.1 for conversion).

6.4.2.2 <u>Estimated errors</u> ($\sigma_{\Delta\phi}$, $\sigma_{\Delta\lambda}$, $\sigma_{\Delta h}$). The estimated errors in a direct WGS 72 to WGS 84 transformation are

$$\sigma_{\Delta\phi} = \sigma_{\Delta\lambda} = 3 \text{ m} \tag{6.11}$$

$$\sigma_{\Delta h} = 3 \text{ m to } 4 \text{ m}$$

These error estimates do not include errors in the original WGS 72 coordinates. The accuracy estimate for $\sigma_{\Delta h}$ reflects discrepancies between WGS 72 and WGS 84 geoid models.

6.4.3 <u>Two-step WGS 72 to WGS 84 transformation</u>. If WGS 72 coordinates are known to have been derived from local geodetic system coordinates using WGS 72 ΔX, ΔY, and ΔZ datum shift parameters, the preferred method for transforming them to the WGS 84 system is to do a two-step conversion. First, do a WGS 72 to local transformation using the inverse of the trans-formation originally used to shift local datum coordinates to WGS 72. This step will eliminate the inaccuracies introduced using local to WGS 72 datum shifts. Next, do a local geodetic sys-tem to WGS 84 conversion to produce coordinates within the WGS 84 system (see Sections 6.2, or 6.3). Seppelin (1974) provides mean value local datum to WGS 72 datum shift parameters. Starting with NGA TR 8350.2 (1997), the year of initial publication and cycle numbers are pro-vided to allow the user to determine when a set of transformation parameters was initially pub-lished and whether the current set replaces an outdated set. A cycle number of zero indicates the set has not changed since the first time parameters for that datum were published. The cycle number is incremented by one for each change. For example, in 1997 NIMA used better infor-mation to create a new set of transformation parameters between WGS 84 and the Tokyo Datum for use in Korea. There are now two sets of transformation parameters that use the identifier "TOY-B" but they have different cycle numbers, "0" and "1."

6.5 <u>Approximating geodetic heights for datum Transformation</u>. When doing the Molodensky approximation (Section 6.3.3), ellipsoidal height (h) is often unavailable. MSL height (H) can be substituted for h without introducing errors in horizontal position significant to mapping and charting applications. The impact of this substitution is so small that zero can be used in place of h, if H is unavailable.

Note: The generation and usage of local datum ellipsoidal heights are outside the scope of this handbook. For such applications consult TEC.

6.6 <u>Multiple regression equations (MRE)</u>. The use of MREs for datum shifting is not recommended.

6.7 Vertical datum shifts. Because exact relationships between vertical datums are unknown, shifting elevations between different vertical datums should not be attempted, except for WGS 72 elevations computed from measured WGS 72 geodetic heights (see Section 6.7.1.) Otherwise, the following are assumed to be the same:

   a. Elevations on different local vertical datums.

   b. Elevations computed from measured WGS 84 geodetic heights and WGS 84 geoid separations.

It is estimated that errors resulting from this assumption will be one to two meters. Should the accuracy be insufficient, contact TEC for assistance.

6.7.1 Satellite-derived WGS 72 elevations based on measured WGS 72 geodetic heights. The WGS 84 geoid model is a closer approximation to the geoid than the older WGS 72 geoid model. Elevations obtained from WGS 72 geodetic height measurements and geoid separations may be improved by converting to WGS 84 using

$$H = H_{WGS\ 72} + N_{WGS\ 72} + \Delta h - N_{WGS\ 84} \qquad\qquad (6.12)$$

where: $H_{WGS\ 72}$ is the elevation computed using the WGS 72 geoid, $N_{WGS\ 72}$ is the WGS 72 geoid separation, and Δh is computed using equation set (6.9).

6.7.2 Survey-derived elevations. Elevations obtained by conventional leveling surveys, for points with WGS 72 geodetic horizontal coordinates, should not be converted to WGS 84 elevations. Simplified procedures to convert WGS 72-derived elevations to WGS 84 are not recommended.

6.8 Unlisted datums. This handbook provides datum shift parameters between many local datums and WGS 84. Regional shift parameters also are given for portions of many of these datums. Contact TEC for any datums and their respective shift parameters not listed in this handbook.

7   MAP PROJECTIONS

7.1 Introduction. Map projections are used to represent the earth's features or points on a map or in a plane coordinate system. A map projection is a systematic representation of a part of the earth's surface on a plane.

7.1.1 The map projection process. The map projection process can be done in three steps:

a. The positions of points on the surface of the earth (ϕ, λ, h) are reduced to positions on the reference ellipsoid (ϕ, λ, h = 0).

b. Next, a map projection is chosen and used to transform the feature's geodetic coordinates (ϕ, λ, h = 0) into planar map projection coordinates (x, y).

c. Finally, the map projection coordinates of the features may be plotted, on a map, or stored for future use.

7.1.2 Properties of projections. Once the earth's surface is projected on a plane, one can observe such properties as distance, angle, direction, shape, and size. None of the projections described in this handbook faithfully preserve all these characteristics. Some type of distortion is always introduced by a map projection; however, a given map projection can preserve some of these properties, and if a sufficiently small part of the earth's surface is displayed, then other properties can be represented approximately. Map projections presented in this handbook have areas of coverage beyond which the projection should not be used due to increased distortion.

7.1.3 Mapping equations. The map projections discussed in this handbook have mapping equations that are defined on a reference ellipsoid and are functions of geodetic coordinates (ϕ, λ). Geodetic height, h, does not enter into the projection equations since h = 0 on a reference ellipsoid. The equations that express projection coordinates, x and y, in terms of (ϕ, λ) are sometimes called **mapping equations**.

7.1.4 Conformal projection. Each projection presented in this handbook has the important property of being conformal. A **conformal projection** is angle preserving. If two lines on the ellipsoid meet at an angle of θ, then in a conformal projection the image of these lines on the map meet at the same angle, θ.

7.1.5 Scale factor. Two quantities associated with map projections, which are often confused, are the scale factor of the projection and the map scale. The **scale factor** of the projection is used in precise direction and distance calculations using a map projection's coordinates. The scale factor of a projection is the ratio of arc length along a differentially small line in the plane of the projection to arc length on the ellipsoid. This number depends on both

of the projection to arc length on the ellipsoid. This number depends on both the location of the point and on the direction of the line along which arc length is being measured; however, for conformal projections, the scale factor of the projection is independent of the direction of the line and depends only upon the location of the point. Since all projections treated in this handbook are conformal, the scale of the projection will be referred to as the point scale factor (k).

7.1.6 Map scale. The **map scale** is the approximate number for converting map distances to terrestrial distances and is usually published on the map. Technically, the map scale is the ratio of a distance on the map to the corresponding distance on the ellipsoid. Although the map scale changes over the area of a map sheet, it is customary to assign a single value to denote the map's scale. This value is the true map scale along at least one line on the map. Usually, the map scale is given by a ratio 1:n, so that the larger the map scale, the smaller the value of n. For military maps of scale greater than 1:100,000 (n < 100,000), the map scale changes only slightly over a map sheet, and the map scale approximates the ratio of distance on the map to the distance on the ellipsoid.

7.1.7 Convergence of the meridian. The **convergence of the meridian** (γ), often called grid convergence, is used to convert between direction in the map projection (grid azimuth) and geodetic azimuth. The convergence of the meridian at a point on a map is the angle between the projection of the meridian at that point and the projection North axis (typically represented by the y axis of the projection). The convergence of the meridian is positive in the clockwise direction. Convergence of the meridian should not be confused with declination, which is the difference between geodetic and magnetic north at the point.

7.1.8 Information sources. Information on map projections can be found in Krakiwsky (1973); NSWC/DL TR-3624 (1977); U.S. Department of Commerce, Coast and Geodetic Survey, Special Publication No. 251 (1979); and Geological Survey Professional Paper 1395 (1987).

7.2 Mercator projection. The Mercator projection is a conformal projection for which the point scale factor is one along the equator. The equator lies on the line y = 0. This projection is not defined at the poles.

7.2.1 Meridians and parallels. Meridians and parallels provide the framework for the Mercator projection. Meridians are projected as parallel straight lines that satisfy the equation x = constant. Evenly spaced meridians on the ellipsoid project to evenly spaced straight lines on the projection. Parallels are projected as parallel straight lines perpendicular to meridians and satisfy the equation y = constant. Evenly spaced parallels on the ellipsoid project to unevenly spaced parallels on the projection. The spacing between projected parallels increases with distance from the equator. See Figure 7.1.
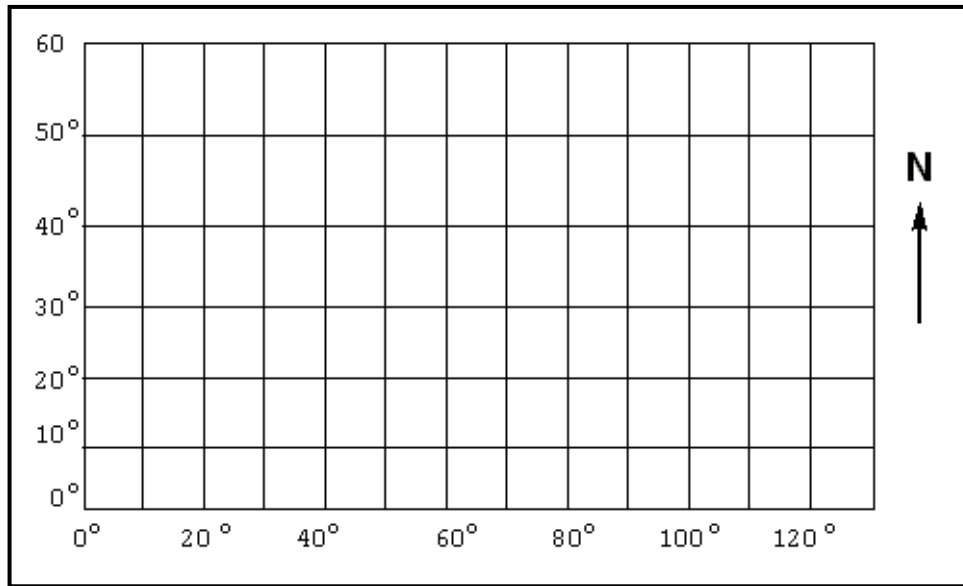
Figure 7.1. Meridians (longitude) and parallels (latitude) in the Mercator projection.

7.2.2 Mercator mapping equations.

7.2.2.1 Finding (x,y). The following discussion of the Mercator mapping equations can be found in Geological Survey Professional Paper 1395 (1987). First, choose a central meridian (λ_0) that represents the zero point on the x axis. Given a point (ϕ, λ), in radians, on the ellipsoid, the corresponding point (x, y) in the plane is

$$x = a(\lambda - \lambda_0) \tag{7.1}$$

$$y = a \ln\left[\tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\left(\frac{1 - \varepsilon \sin\phi}{1 + \varepsilon \sin\phi}\right)^{\frac{\varepsilon}{2}} \right]$$

or

$$y = \left(\frac{a}{2}\right) \ln\left[\left(\frac{1 + \sin\phi}{1 - \sin\phi}\right)\left(\frac{1 - \varepsilon \sin\phi}{1 + \varepsilon \sin\phi}\right)^{\varepsilon} \right] \tag{7.2}$$

where

  a = semi-major axis of the ellipsoid

  ε = eccentricity

  λ_0 = longitude of the central meridian

ϕ, λ and λ_0 are expressed in radians.

7.2.2.2 <u>Point scale factor and convergence of the meridian</u>. The equations for the point scale factor (k) and the convergence of the meridian (γ) are

$$k = \frac{a}{N \cos \phi} \qquad (7.3)$$

$$\gamma = 0$$

where

$$N = \frac{a}{\sqrt{1 - \varepsilon^2 \sin^2 \phi}}$$

7.2.2.3 <u>Finding (ϕ, λ)</u>. Next, consider the inverse problem of finding (ϕ, λ) when (x, y) is known. The longitude can be found from

$$\lambda = \frac{x}{a} + \lambda_0 \qquad (7.4)$$

where both λ and λ_0 are expressed in radians. The value of ϕ can be found iteratively. Let an intermediate parameter t be defined

$$t = e^{\frac{-y}{a}} \qquad (7.5)$$

After an initial value of

$$\phi_0 = \frac{\pi}{2} - 2 \arctan t \qquad (7.6)$$

iterate the following equation for ϕ:

$$\phi_{n+1} = \frac{\pi}{2} - 2 \arctan\left[t \left(\frac{1 - \varepsilon \sin\phi_n}{1 + \varepsilon \sin\phi_n} \right)^{\frac{\varepsilon}{2}} \right] \qquad (7.7)$$

Substitute the initial value (ϕ_0) into equation 7.7 to find the next candidate ϕ_1. Similarly, substitute ϕ_1 into equation 7.7 to obtain the updated candidate, ϕ_2. Continue this process until the difference between successive values of ϕ_n is sufficiently small. Then set $\phi = \phi_n$. All angles are measured in radians.

7.2.3 <u>Accuracy</u>. The Mercator equations for x, y, k, γ, and λ are exact. The iterative equation for ϕ can be updated until any desired accuracy is obtained.

7.2.4 Area of coverage. In the Mercator projection, as the latitude (ϕ) approaches the poles, the y coordinate approaches infinity. Area and length distortion increases with distance from the equator. For example, the point scale factor is approximately 2 at 60° latitude and 5.7 at 80° latitude.

7.3 Transverse Mercator (TM) projection. The Transverse Mercator projection is a conformal projection for which the point scale factor equals one along the central meridian. The line y = 0 is the projection of the equator, and the line x = 0 is the projection of the central meridian. The Universal Transverse Mercator (UTM) projection (see Section 7.4) is a modification of the Transverse Mercator projection.

7.3.1 Meridians and parallels. Both the central meridian and the equator are represented as straight lines. No other meridian or parallel is projected onto a straight line. Since the point scale factor is one along the central meridian, this projection is most useful near the central meridian. Scale distortion increases away from this meridian (see Figure 7.2.).
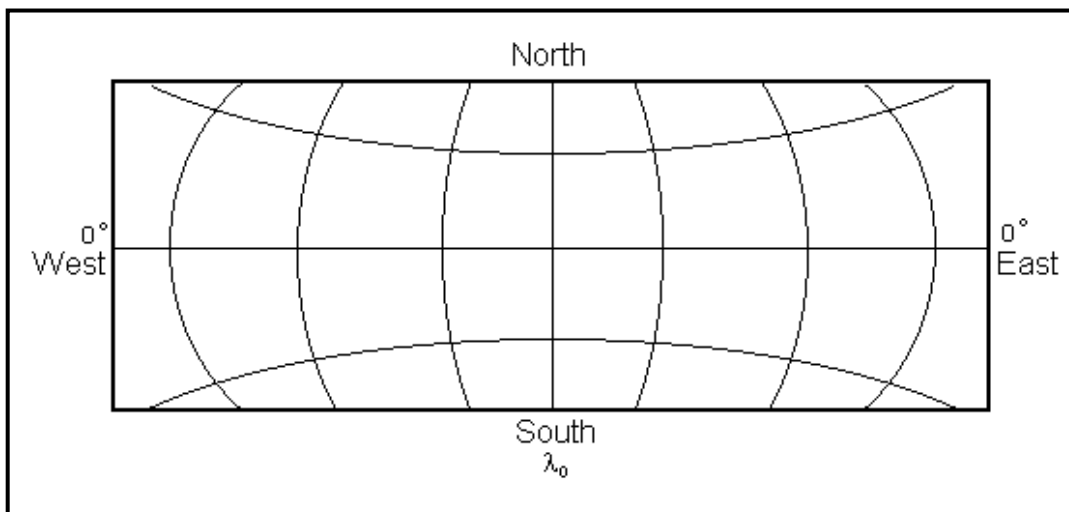
Figure 7.2. Meridians and parallels in the Transverse Mercator Projection. λ_0 is the central meridian.

7.3.2 Transverse Mercator mapping equations.

7.3.2.1 Finding (x, y). The presentation here follows U.S. Department of Commerce, Coast and Geodetic Survey, Special Publication No. 251 (1979). A point with geodetic coordinates (ϕ, λ) in radians has Transverse Mercator coordinates (x, y) that are given by

$$x = N \Lambda \cos \phi + \frac{N \Lambda^3 \cos^3 \phi}{6}(1 - t^2 + \eta^2) + \frac{N \Lambda^5 \cos^5 \phi}{120}(5 - 18t^2 + t^4 + 14\eta^2 - 58t^2 \eta^2) \quad (7.8)$$

$$y = S_\phi + \frac{N \Lambda^2}{2} \sin \phi \cos \phi + \frac{N \Lambda^4}{24} \sin \phi \cos^3 \phi \, (5 - t^2 + 9\eta^2 + 4\eta^4)$$

$$+ \frac{N \Lambda^6}{720} \sin \phi \cos^5 \phi \, (61 - 58t^2 + t^4 + 270\eta^2 - 330t^2 \eta^2)$$

where

$$N = \frac{a}{\sqrt{1 - \varepsilon^2 \sin^2 \phi}} \tag{7.9}$$

$$t = \tan \phi$$

$$\eta = \varepsilon' \cos \phi$$

$$\varepsilon'^2 = \frac{\varepsilon^2}{(1 - \varepsilon^2)}$$

$$\varepsilon^2 = \frac{a^2 - b^2}{a^2}$$

$$S_\phi = a \, (A_0\phi - A_2 \sin 2\phi + A_4 \sin 4\phi - A_6 \sin 6\phi + A_8 \sin 8\phi)$$

$$A_0 = 1 - \frac{1}{4} \varepsilon^2 - \frac{3}{64} \varepsilon^4 - \frac{5}{256} \varepsilon^6 - \frac{175}{16384} \varepsilon^8$$

$$A_2 = \frac{3}{8} \, (\varepsilon^2 + \frac{1}{4} \varepsilon^4 + \frac{15}{128} \varepsilon^6 - \frac{455}{4096} \varepsilon^8)$$

$$A_4 = \frac{15}{256} \, (\varepsilon^4 + \frac{3}{4} \varepsilon^6 - \frac{77}{128} \varepsilon^8)$$

$$A_6 = \frac{35}{3072} \, (\varepsilon^6 - \frac{41}{32} \varepsilon^8)$$

$$A_8 = - \frac{315}{131072} \varepsilon^8$$

$$\Lambda = \lambda - \lambda_0 \text{ in radians}$$

$$\lambda_0 = \text{central meridian in radians}$$

At the poles, where $\phi = \pm \frac{\pi}{2}$, the Transverse Mercator coordinates are x = 0 and y = S_ϕ, a quadrant arc on the ellipsoid.

7.3.2.2 Finding (ϕ, λ).

7.3.2.2.1 Finding the footpoint latitude (ϕ_1). To find (ϕ, λ), given x and y, first compute the footpoint latitude (ϕ_1) using the following iterative method (See Krakiwsky 1973). The equation to be iterated to yield the footpoint latitude (ϕ_1) is

$$\phi' = \phi - \frac{S_\phi - y}{S'_\phi} \tag{7.10}$$

where

$$S'_\phi = a\,(A_o - 2A_2\cos2\phi + 4A_4\cos4\phi - 6A_6\cos6\phi + 8A_8\cos8\phi) \tag{7.11}$$

and S_ϕ, A_0, A_2, A_4, A_6, and A_8 are defined in equation set 7.9.

Let the first candidate for ϕ_1 be $\phi = y/a$. When this value of ϕ is substituted into equation 7.10, the updated candidate for ϕ_1 is ϕ'. Similarly, the substitution of ϕ' for ϕ in equation 7.10 gives the next updated value of ϕ'. When successive values of ϕ' are sufficiently close, then the footpoint latitude is found by setting $\phi_1 = \phi'$. A non-iterative method for finding ϕ_1 can be found in NOAA Report NOS NGS 5 (1989).

7.3.2.2.2 Finding (ϕ, λ) from ϕ_1. Once ϕ_1 is determined, (ϕ, λ) can be found from the approximations

$$\phi = \phi_1 - \frac{t_1\,N_1}{R_1}\left[\frac{1}{2}\left(\frac{x}{N_1}\right)^2 - \frac{B_4}{24}\left(\frac{x}{N_1}\right)^4 + \frac{B_6}{720}\left(\frac{x}{N_1}\right)^6\right] \tag{7.12}$$

$$\lambda = \lambda_0 + \sec\phi_1\left[\frac{x}{N_1} - \frac{B_3}{6}\left(\frac{x}{N_1}\right)^3 + \frac{B_5}{120}\left(\frac{x}{N_1}\right)^5\right]$$

where

$$B_3 = 1 + 2t_1^2 + \eta_1^2 \tag{7.13}$$

$$B_4 = 5 + 3t_1^2 + \eta_1^2 - 4\eta_1^4 - 9t_1^2\,\eta_1^2$$

$$B_5 = 5 + 28t_1^2 + 24t_1^4 + 6\eta_1^2 + 8t_1^2\,\eta_1^2$$

$$B_6 = 61 + 90t_1^2 + 46\eta_1^2 + 45t_1^4 - 252t_1^2\,\eta_1^2 - 3\eta_1^4 - 66t_1^2\,\eta_1^4 - 90t_1^4\,\eta_1^2 + 225t_1^4\,\eta_1^4$$

$$\varepsilon'^2 = \frac{\varepsilon^2}{(1 - \varepsilon^2)}$$

$$\eta_1^2 = \varepsilon'^2 \cos^2 \phi_1$$

$$t_1 = \tan \phi_1$$

$$N_1 = \frac{a}{\sqrt{1 - \varepsilon^2 \sin^2 \phi_1}}$$

$$R_1 = \frac{a(1 - \varepsilon^2)}{(1 - \varepsilon^2 \sin^2 \phi_1)^{3/2}}$$

If $\phi_1 = \pm \dfrac{\pi}{2}$ then $\phi = \pm 90°$ (a pole) and λ is undefined.

7.3.2.3 <u>Point scale factor as a function of x and ϕ_1</u>. As a function of x and ϕ_1, the point scale factor is given by:

$$k = 1 + \frac{C_2}{2}\left(\frac{x}{N_1}\right)^2 + \frac{C_4}{24}\left(\frac{x}{N_1}\right)^4 + \frac{1}{720}\left(\frac{x}{N_1}\right)^6 \tag{7.14}$$

where

$$C_2 = 1 + \eta_1^2 \tag{7.15}$$

$$C_4 = 1 + 6\eta_1^2 + 9\eta_1^4 - 24t_1^2 \eta_1^2$$

and t_1, η_1, and N_1 are given by equation set 7.13.

7.3.2.4 <u>Point scale factor as a function of (ϕ,λ)</u>. As a function of (ϕ, λ), the point scale factor is given by

$$k = 1 + \frac{\Lambda^2 \cos^2 \phi}{2} D_2 + \frac{\Lambda^4 \cos^4 \phi}{24} D_4 + \frac{\Lambda^6 \cos^6 \phi}{720} D_6 \tag{7.16}$$

in which

$$D_2 = 1 + \eta^2 \tag{7.17}$$

$$D_4 = 5 - 4t^2 + 14\eta^2 + 13\eta^4 - 28t^2\eta^2 - 48t^2\eta^4$$

$$D_6 = 61 - 148t^2 + 16t^4$$

and Λ, t, and η are given in equation set 7.9.

7.3.2.5 <u>Convergence of the meridian in terms of x and ϕ_1</u>. The convergence of the
meridian, in terms of x and ϕ_1, is

$$\gamma = \frac{t_1\, x}{N_1} \left[1 - \frac{E_2}{3} \left(\frac{x}{N_1} \right)^2 + \frac{E_4}{15} \left(\frac{x}{N_1} \right)^4 - \frac{E_6}{315} \left(\frac{x}{N_1} \right)^6 \right] \qquad (7.18)$$

where

$$E_2 = 1 + t_1^2 - \eta_1^2 - 2\eta_1^4 \qquad (7.19)$$

$$E_4 = 2 + 5t_1^2 + 2\eta_1^2 + 3t_1^4 + t_1^2\, \eta_1^2 + 9\eta_1^4 - 7\, t_1^2\, \eta_1^4$$

$$E_6 = 17 + 77t_1^2 + 105t_1^4 + 45t_1^6$$

and t_1, η_1, and N_1 are given by equation set 7.13.

7.3.2.6 <u>Convergence of the meridian in terms of (ϕ, λ)</u>. The convergence of the meridian,
in terms of (ϕ, λ), is

$$\gamma = \Lambda\, \sin\, \phi \left(1 + F_2 \frac{\Lambda^2\, \cos^2\, \phi}{3} + F_4 \frac{\Lambda^4\, \cos^4\, \phi}{15} + F_6 \frac{\Lambda^6\, \cos^6\, \phi}{315} \right) \qquad (7.20)$$

in which

$$F_2 = 1 + 3\eta^2 + 2\eta^4 \qquad (7.21)$$

$$F_4 = 2 - t^2 + 15\eta^2 + 35\eta^4 - 15t^2\eta^2 - 50t^2\eta^4$$

$$F_6 = 17 - 26t^2 + 2t^4$$

and Λ, t, and η are given in equation set 7.9.

7.3.3 <u>Accuracy</u>. The Transverse Mercator equations for x, y, ϕ, λ, S_ϕ, ϕ_1, k, and γ are
approximations. Within 4° of the central meridian, the equations for x, y, ϕ, and λ have an error
of less than 1 centimeter (see Geological Survey Professional Paper 1395, 1987).

7.3.4 <u>Area of coverage</u>. The equations given here for the Transverse Mercator projection
can be used within 4° of the central meridian. If these equations are used with UTM coordinates,
then consult Section 7.4.1 for guidance on the area of coverage for UTM coordinates.

7.4 Universal Transverse Mercator (UTM) projection.   The UTM projection is a family of projections that differ from the Transverse Mercator projection in these ways (see Figure 7.3):

a. The longitudes of the central meridians (in degrees) are 3 + 6n, n = 0,...59.

b. The point scale factor along the central meridian is 0.9996.

c. The y value, called the northing, has an origin of 0 meters in the northern hemisphere at the equator. The y value, called the northing, has an origin of 10,000,000 meters at the equator and decreases toward the pole in the southern hemisphere.

d. The x value, called the easting, has an origin of 500,000 meters at the central meridian.

7.4.1 UTM zones. For the UTM grid system, the ellipsoid is divided into 60 longitudinal zones of 6 degrees each. Zone number one lies between 180° E and 186° E. The zone numbers increase consecutively in the eastward direction. It is intended that a UTM projection should be used only in one of the 6-degree zones, plus the overlap area. In each zone, the UTM projection extends for a 40 km overlap into the two adjacent zones. Moreover, UTM coordinates are used only between 80° S and 84° N plus the overlap region. The UTM projection extends to 84° 30′ N and 80° 30′ S to overlap with the Universal Polar Stereographic (UPS) projections.

7.4.1.1 Finding the UTM zone and central meridian. To find UTM coordinates from geodetic coordinates, first compute the UTM zone. A standard zone can be computed directly from geodetic longitude. If the geodetic longitude (λ) is expressed in positive radians, then the zone (z) is

$$z = \text{Greatest Integer} \leq \left(31 + \frac{180\lambda}{6\pi} \right) \text{ for } 0 \leq \lambda < \pi \qquad (7.22)$$

$$z = \text{Greatest Integer} \leq \left(\frac{180\lambda}{6\pi} - 29 \right) \text{ for } \pi \leq \lambda < 2\pi$$
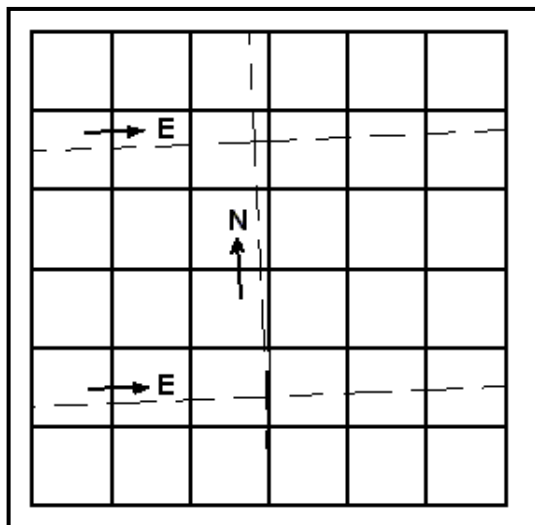
Figure 7.3. Meridians and parallels (dashed) and a Universal Transverse Mercator Grid.

Once the zone (z) is known, the central meridian is given by equation set 7.23. If a longitude lies on a UTM zone boundary, then the user must choose the desired UTM zone. If a map is being used, then the UTM zone appears on the map. The central meridian of zone (z) is

$$\lambda_0 = (6z - 183)\frac{\pi}{180} \quad \text{for } z \geq 31 \tag{7.23}$$

$$\lambda_0 = (6z + 177)\frac{\pi}{180} \quad \text{for } z \leq 30$$

with λ_0 expressed in radians

7.4.1.2 Non-standard width UTM zones. Non-standard width UTM zones are used in North Atlantic regions and Norway as delineated in Table 7.1 (see DMA TM 8358.1 (1990). These non-standard zones also may be applied in the 30′ UPS overlap region.

Table 7.1. Non-standard zone limits.

Zone	Latitude Lower	Latitude Upper	Longitude West	Longitude East	Longitude Central Meridian
31	56° N	64° N	0° E	3° E	3° E
32	56	64	3	12	9
31	72	84	0	9	3
32	72	84		not used	
33	72	84	9	21	15
34	72	84		not used	
35	72	84	21	33	27
36	72	84		not used	
37	72	84	33	42	39

7.4.2 Reference source. The standard references for the UTM projection are DMA TM 8358.1 (1990) and DMA TM 8358.2 (1989).

7.4.3 UTM equations.

7.4.3.1 Finding UTM coordinates (x_{UTM}, y_{UTM}). Having found the central meridian, next compute Transverse Mercator coordinates (x, y), using equation sets 7.8 and 7.9. Then, the UTM coordinates can be found from:

$$x_{UTM} = 0.9996\ x_{TM} + 500,000 \tag{7.24}$$

$$y_{UTM} = 0.9996\ y_{TM}\ \text{(Northern hemisphere)}$$

$$y_{UTM} = 0.9996\ y_{TM} + 10,000,000\ \text{(Southern hemisphere)}$$

$$k_{UTM} = 0.9996\ k_{TM}$$

$$\gamma_{UTM} = \gamma_{TM}$$

7.4.3.2 Computing geodetic coordinates. To compute geodetic coordinates from UTM coordinates, a zone, and a hemisphere, first solve equation set 7.25 for (x_{TM}, y_{TM}), equation set 7.23 for λ_0, and then compute (ϕ, λ) from equation set 7.12.

$$x_{TM} = \frac{1}{0.9996}\ (x_{UTM} - 500,000) \tag{7.25}$$

$$y_{TM} = \frac{1}{0.9996}\ (y_{UTM}) \qquad \text{(Northern hemisphere)}$$

$$y_{TM} = \frac{1}{0.9996}\ (y_{UTM} - 10,000,000) \qquad \text{(Southern hemisphere)}$$

$$k_{TM} = \frac{1}{0.9996}\ (k_{UTM})$$

$$\gamma_{TM} = \gamma_{UTM}$$

7.4.3.3 Discontinuity. Notice that UTM coordinates are discontinuous in y at the equator.

7.4.4 Accuracy. The accuracy of the UTM equations is based on the accuracy of the Transverse Mercator equations. Within the area of coverage, the equations for x, y, ϕ, and λ have an error of less than one centimeter. (See Geological Survey Professional Paper 1395, 1987.)

7.4.5 Area of coverage. The area of coverage for UTM coordinates is defined by zone limits, latitude limits, and overlap.

Zone limits: 6° zones, extending 3° to each side of the central meridian. See Table 7.1 for exceptions to this general rule.

Latitude limits: North: 84°
                    South: 80°

Zone overlap: 40 km on either side of the zone limits.

Polar overlap: 30′ toward the poles
                    North: 84° 30′
                    South: 80° 30′

7.5 Lambert Conformal Conic projection. The Lambert conformal conic projection is a conformal projection in which the projected parallels are arcs of concentric circles centered at the pole. The projected meridians are radii of concentric circles that meet at the pole. Moreover, there are one or two parallels, called standard parallels, along which the point scale factor is one (see Figure 7.4.).
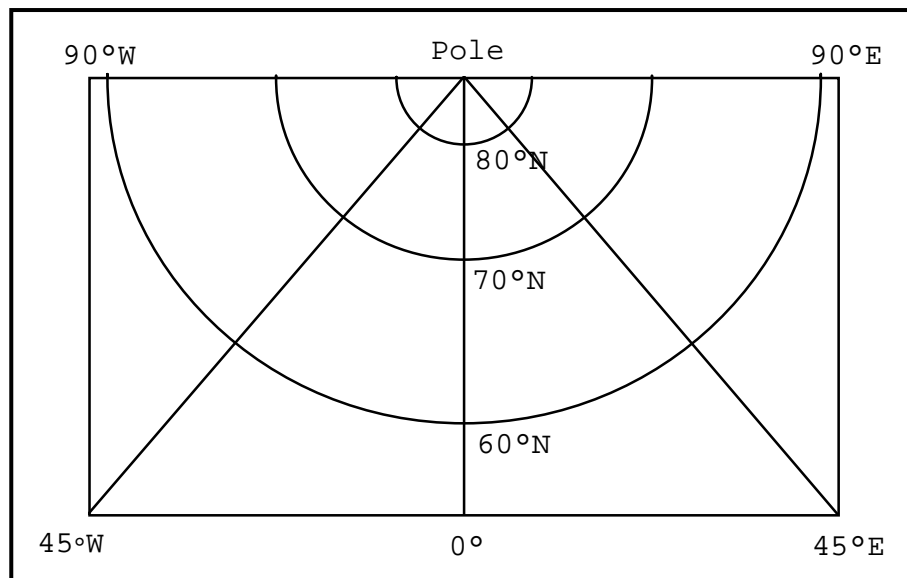

Figure 7.4. Meridians and parallels in the Lambert Conformal Conic Projection (Not drawn to scale).

7.5.1 Lambert Conformal Conic with two standard parallels. This presentation of the mapping equations follows Krakiwsky (1973). The first case to be considered is the case of two standard parallels, ϕ_1 and ϕ_2. The point scale factor is one along both standard parallels. Choose a fixed central meridian, λ_0.

7.5.1.1 Finding (x,y). Begin by fixing a latitude (ϕ_0) that is below the area of interest. For a given (ϕ, λ) the mapping equations are

$$x = r \sin L\Lambda \tag{7.26}$$

$$y = r_0 - r \cos L\Lambda$$

where

$$\Lambda = \lambda - \lambda_0$$

$$r = K e^{-Lq} \tag{7.27}$$

$$r_0 = K e^{-Lq_0}$$

$$K = \frac{N_1 \cos \phi_1}{L e^{-Lq_1}} = \frac{N_2 \cos \phi_2}{L e^{-Lq_2}}$$

e = base of natural logarithms (ln e = 1)

ϕ_1 = first standard parallel

ϕ_2 = second standard parallel

$$N = \frac{a}{\sqrt{1 - \varepsilon^2 \sin^2 \phi}}$$

$$q = \ln \left[\tan \left(\frac{\pi}{4} + \frac{\phi}{2} \right) \left(\frac{1 - \varepsilon \sin \phi}{1 + \varepsilon \sin \phi} \right)^{\frac{\varepsilon}{2}} \right]$$

$$\varepsilon^2 = \frac{a^2 - b^2}{a^2}$$

$$L = \frac{\ln (N_1 \cos \phi_1) - \ln (N_2 \cos \phi_2)}{q_2 - q_1}$$

q_j = q evaluated at ϕ_j , j = 0,1,2

N_j = N evaluated at ϕ_j , j = 1,2

7.5.1.2 Finding (ϕ,λ) with two standard parallels. For the reverse case, when (x, y) is given along with ϕ_0, ϕ_1, ϕ_2, and λ_0, first compute L, r_0, K, N_1, N_2, q_0, q_1, and q_2 from equation set 7.27. Then compute λ and q from

$$\lambda = \frac{\theta}{L} + \lambda_0 \tag{7.28}$$

$$q = \frac{\ln \frac{K}{r}}{L}$$

in which

$$r = \frac{r_0 - y}{\cos \theta}$$

$$0 < y < r_0 \tag{7.29}$$

$$\theta = \arctan \frac{x}{r_0 - y}$$

Once q is found, then ϕ can be determined using the Newton - Raphson method, which is explained next. Define functions f and f′, and an initial approximation ϕ′ by

$$f(\phi) = -q + \frac{1}{2} \ln \left[\left(\frac{1 + \sin \phi}{1 - \sin \phi} \right) \left(\frac{1 - \varepsilon \sin \phi}{1 + \varepsilon \sin \phi} \right)^{\varepsilon} \right] \tag{7.30}$$

$$f'(\phi) = \frac{1 - \varepsilon^2}{(1 - \varepsilon^2 \sin^2 \phi)\cos \phi}$$

$$\phi' = 2 \arctan (e^q) - \pi/2$$

Then successive iterations for ϕ are found from the equation

$$\phi'' = \phi' - \frac{f(\phi')}{f'(\phi')} \tag{7.31}$$

In particular, substitute ϕ′ into this equation to obtain ϕ″, the first approximation to ϕ. Then set ϕ′ = ϕ″, substitute ϕ′ into the equation to obtain ϕ″, the second approximation to ϕ. Continue this procedure until the differences between successive candidates for ϕ are sufficiently small.

7.5.2 Lambert Conformal Conic with one standard parallel. In the second case, the point scale factor is one along one standard parallel, ϕ_0. Fix a central meridian, λ_0.

7.5.2.1 Finding (x, y). The mapping equations are

$$x = r \sin L\Lambda \tag{7.32}$$

$$y = r_0 - r \cos L\Lambda$$

where

$$\Lambda = \lambda - \lambda_0$$

$$L = \sin \phi_0 \tag{7.33}$$

$$K = N_0 \cot \phi \; e^{q_0 \sin\phi_0}$$

$$r = K \, e^{-Lq}$$

$$r_0 = K \, e^{-Lq_0}$$

e = base of natural logarithms (ln e = 1),

$$N = \frac{a}{\sqrt{1 - \varepsilon^2 \sin^2 \phi}}$$

$$q = \ln \left[\tan \left(\frac{\pi}{4} + \frac{\phi}{2} \right) \left(\frac{1 - \varepsilon \sin \phi}{1 + \varepsilon \sin \phi} \right)^{\frac{\varepsilon}{2}} \right]$$

$q_0 = q$ evaluated at ϕ_0

$N_0 = N$ evaluated at ϕ_0

7.5.2.2 Finding (ф,λ) with one standard parallel. To transform in the other direction, suppose that x, y, ϕ_0, and λ_0 are given. First, compute L, K, N_0, q_0, r, and r_0 from equation set 7.33. Then, as in the previous case, (ф, λ) can be found from

$$\lambda = \frac{\theta}{L} + \lambda_0 \tag{7.34}$$

$$q = \frac{\ln \dfrac{K}{r}}{L}$$

in which

$$r = \frac{r_0 - y}{\cos \theta} \tag{7.35}$$

$$\theta = \arctan \frac{x}{r_0 - y}$$

Again, ϕ can be obtained from q by using the Newton-Raphson method (see equation sets 7.30 and 7.31).

7.5.3 Scale factor and convergence. The scale factor and convergence of the meridian are given by

$$k = \frac{rL}{N \cos \phi} \tag{7.36}$$

$$\gamma = L\Lambda$$

In the case of two standard parallels, the values of r, L, and N are given in equation set 7.27. For one standard parallel, the values of r, L, and N are found in equation set 7.33.

7.5.4 Accuracy. For the Lambert conformal conic projection with either one or two standard parallels, the equations for x, y, λ, q, k, and γ are exact. The equation for ϕ can be iterated until the desired accuracy is obtained.

7.5.5 Area of coverage. The family of Lambert conformal conic projections and their limiting cases is used worldwide. For a given member of this family, the point scale factor increases as a point moves away from the standard parallel(s).

7.6 Polar Stereographic projection. The Polar Stereographic projection is a limiting case of the Lambert conformal conic projection when the one standard parallel approaches a pole. In this conformal projection, meridians are straight lines, parallels are concentric circles, and the point scale factor is one at the pole (see Figure 7.5).
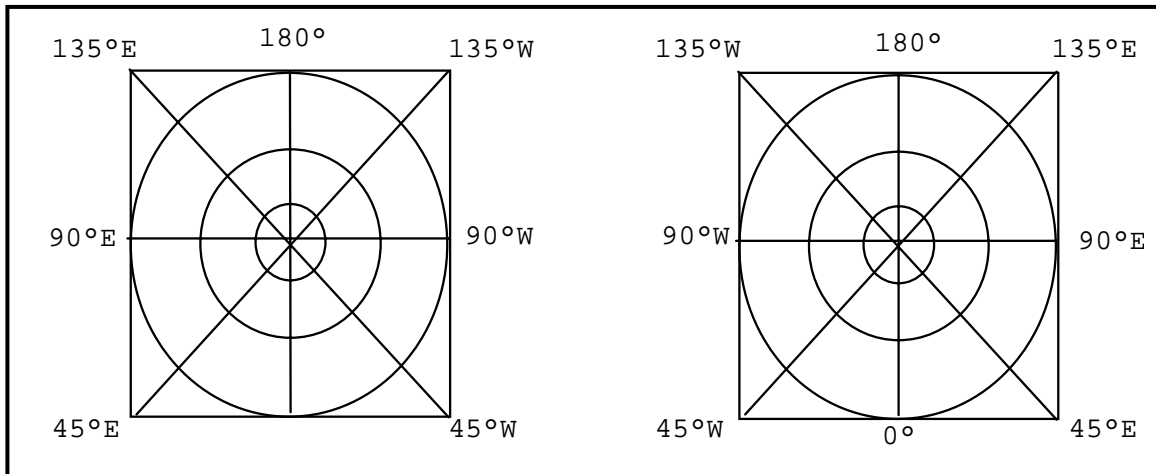
Figure 7.5. Meridians and parallels in the Universal Polar Stereographic Projection, Left: South Zone, Right: North Zone (Not drawn to scale).

7.6.1 Polar Stereographic mapping equations. There is considerable diversity in the treatment that various authors give to the polar Stereographic projection. Some authors interchange the x and y axis; others change the sign convention. The discussion here is largely taken from NSWC/DL TR-3624 (1977) since it most closely conforms to DMA TM 8358.2 (1989).

7.6.1.1 Finding (x,y). Given (ϕ, λ), the Polar Stereographic coordinates are

$$x = r \sin \lambda \qquad\qquad (7.37)$$

$$y = -r \cos \lambda \quad \text{(Northern hemisphere)}$$

$$y = r \cos \lambda \quad \text{(Southern hemisphere)}$$

in which

$$r = K \tan \left(\frac{\pi}{4} - \frac{|\phi|}{2} \right) \left(\frac{1 + \varepsilon \sin |\phi|}{1 - \varepsilon \sin |\phi|} \right)^{\frac{\varepsilon}{2}} \qquad\qquad (7.38)$$

$$K = \frac{2a^2}{b} \left(\frac{1 - \varepsilon}{1 + \varepsilon} \right)^{\frac{\varepsilon}{2}}$$

$$\varepsilon^2 = \frac{a^2 - b^2}{a^2}$$

$|\phi|$ denotes the absolute value of ϕ

ϕ and λ are expressed in radians.

At the poles, λ is not defined. Nevertheless, these equations can be used at the poles since r is zero there. At $\phi = \pm \pi/2$, pick an arbitrary value for λ. Then, equation sets 7.37 and 7.38 yield Polar Stereographic coordinates of (0, 0).

7.6.1.2 Finding (ϕ, λ). The discussion of the conversion from (x, y) to (ϕ, λ) follows Krakiwsky (1973). If (x, y) = (0, 0), then $\phi = \pm 90°$ and λ is undefined. Otherwise, the longitude is given by

$$\lambda = -\arctan\frac{x}{y} \qquad \text{(Northern hemisphere)} \qquad (7.39)$$

$$\lambda = \arctan\frac{x}{y} \qquad \text{(Southern hemisphere)}$$

If (x, y) $\neq$ (0, 0), then x, y or both are non-zero. Use one of these non-zero values to find q from the equations

$$e^{-q} = \frac{|y|}{K\,|\cos \lambda|} \qquad \text{when } |x| > |y| \qquad (7.40)$$

$$e^{-q} = \frac{x}{K\sin \lambda} \qquad \text{when } |y| \geq |x|$$

where

$$K = \frac{2a^2}{b}\left(\frac{1-\varepsilon}{1+\varepsilon}\right)^{\frac{\varepsilon}{2}} \qquad (7.41)$$

As in the Lambert conformal conic projection, ϕ can be found from q using the Newton-Raphson method. Define functions f and f ', and an initial approximation ϕ_0 by

$$f(\phi) = -q + \frac{1}{2}\ln\left[\left(\frac{1+\sin \phi}{1-\sin \phi}\right)\left(\frac{1-\varepsilon \sin \phi}{1+\varepsilon \sin \phi}\right)^{\varepsilon}\right] \qquad (7.42)$$

$$f'(\phi) = \frac{1-\varepsilon^2}{(1-\varepsilon^2 \sin^2 \phi)\cos \phi}$$

$$\phi_0 = 2\arctan e^{q} - \frac{\pi}{2}$$

Then, successive iterations for ϕ are found from the equation

$$\phi_n = \phi_{n-1} - \frac{f(\phi_{n-1})}{f'(\phi_{n-1})} \tag{7.43}$$

In particular, substitute ϕ_0 into this equation to obtain ϕ_1, the first approximation to ϕ. Then substitute ϕ_1 into the equation to obtain ϕ_2, the second approximation to ϕ. Continue this procedure until the differences between successive candidates for ϕ are sufficiently small. Even though q approaches infinity at the poles, this method is satisfactory for many computers. This procedure yields a positive value of ϕ. In the southern hemisphere, replace this positive value of ϕ by $-\phi$.

7.6.1.3 Alternate method for finding ϕ. Alternatively, an approximate equation for ϕ is given in Geological Survey Professional Paper 1395 (1987).

$$\phi = \chi + A\,\sin2\chi + B\,\sin4\chi + C\,\sin6\chi + D\,\sin8\chi \tag{7.44}$$

where

$$\chi = \frac{\pi}{2} - z \quad \text{(in radians)} \tag{7.45}$$

$$\tan\frac{z}{2} = \frac{x}{K\,\sin\lambda} \qquad\qquad x \neq 0$$

$$\tan\frac{z}{2} = \left|\frac{y}{K\,\cos\lambda}\right| \qquad\qquad y \neq 0$$

$$K = \frac{2a^2}{b}\left(\frac{1-\varepsilon}{1+\varepsilon}\right)^{\frac{\varepsilon}{2}}$$

$$A = \frac{1}{2}\varepsilon^2 + \frac{5}{24}\varepsilon^4 + \frac{1}{12}\varepsilon^6 + \frac{13}{360}\varepsilon^8$$

$$B = \frac{7}{48}\varepsilon^4 + \frac{29}{240}\varepsilon^6 + \frac{811}{11520}\varepsilon^8$$

$$C = \frac{7}{120}\varepsilon^6 + \frac{81}{1120}\varepsilon^8$$

$$D = \frac{4279}{161280}\varepsilon^8$$

$$\varepsilon^2 = \frac{a^2 - b^2}{a^2}$$

Since this procedure yields a positive value of ϕ, replace ϕ by $-\phi$ in the southern hemisphere.

7.6.1.4 Finding the point scale factor. The point scale factor is given by:

$$k = 1 \quad \text{at the poles} \tag{7.46}$$

$$k = \frac{r}{N \cos \phi} \quad \text{elsewhere}$$

where

$$r = K \tan\left(\frac{\pi}{4} - \frac{|\phi|}{2}\right)\left(\frac{1 + \varepsilon \sin |\phi|}{1 - \varepsilon \sin |\phi|}\right)^{\frac{\varepsilon}{2}} \tag{7.47}$$

$$K = \frac{2a^2}{b}\left(\frac{1 - \varepsilon}{1 + \varepsilon}\right)^{\frac{\varepsilon}{2}}$$

$$N = \frac{a}{\sqrt{1 - \varepsilon^2 \sin^2 \phi}}$$

7.6.1.5 Finding the convergence of the meridian. The convergence of the meridian is

$$\gamma = \lambda \qquad \text{(Northern hemisphere)} \tag{7.48}$$

$$\gamma = -\lambda \qquad \text{(Southern hemisphere)}$$

7.6.2 Accuracy. For the Polar Stereographic projection, the equations for x, y, λ, and q are exact. The equation for ϕ can be iterated until the desired accuracy is obtained.

7.6.3 Area of coverage. There is no general agreement on the area of coverage for the Polar Stereographic projection. Section 7.7.5 gives the area of coverage for UPS coordinates.

7.7 Universal Polar Stereographic (UPS) projection. The Universal Polar Stereographic (UPS) projection is the standard military grid used in polar regions (see Figure 7.6). The main reference for the UPS grid is DMA TM 8358.2 (1989). The UPS grid is a family of two projections that differ from the Polar Stereographic projection in these ways:

a. Both the x and y values (see Figure 7.6), called easting and northing, respectively, have an origin of 2,000,000 meters.

b. The scale factor at the origin is 0.994.
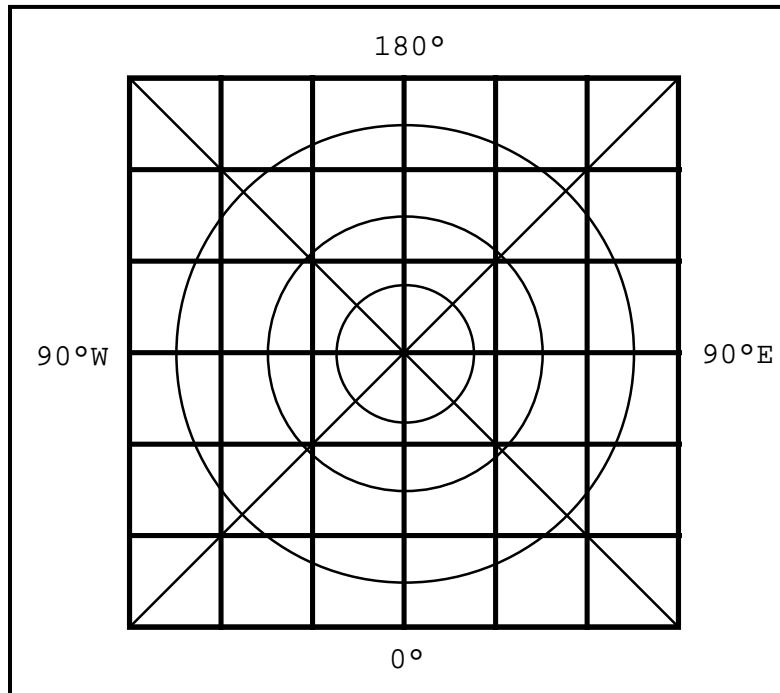
Figure 7.6. Meridians and parallels imposed on a Universal Polar Stereographic grid (north zone).

The limits of the system are north of 84° N and south of 80° S. In order to provide a 30-minute overlap with the UTM grid, the UPS grid contains an overlap that extends to 83° 30′N and 79° 30′S. Although the UPS grid originally was defined to use the International Ellipsoid, in practice, several reference ellipsoids have been used. The ellipsoid appropriate to the selected datum should be used.

7.7.1 Universal Polar Stereographic (UPS) mapping equations. For known geodetic coordinates, the UPS easting is given by

$$x_{UPS} = 2,000,000 + 0.994 \; r \sin \lambda \qquad\qquad (7.49)$$

and the UPS northing is given by

$$y_{UPS} = 2,000,000 - 0.994 \; r \cos \lambda \qquad \text{(Northern hemisphere)} \qquad (7.50)$$

$$y_{UPS} = 2,000,000 + 0.994 \; r \cos \lambda \qquad \text{(Southern hemisphere)}$$

The parameter r is computed using equation set 7.38.

7.7.2 Finding (ϕ,λ). If a hemisphere and UPS coordinates (x, y) are known, convert the UPS values to Polar Stereographic coordinates using equation set 7.51, and then transform to geodetic coordinates as described in Sections 7.6.1.2 and 7.6.1.3.

$$x_{PS} = \frac{1}{0.994}(x_{UPS} - 2,000,000) \qquad\qquad (7.51)$$

$$y_{PS} = \frac{1}{0.994}(y_{UPS} - 2,000,000)$$

In particular, at the poles where $x = 2,000,000$ and $y = 2,000,000$ then, $\phi = \pm 90°$ and λ is undefined. Otherwise longitude and latitude can be found from

$$\lambda = -\arctan\frac{(x_{UPS} - 2,000,000)}{(y_{UPS} - 2,000,000)} \qquad \text{(Northern hemisphere)} \qquad (7.52)$$

$$\lambda = +\arctan\frac{(x_{UPS} - 2,000,000)}{(y_{UPS} - 2,000,000)} \qquad \text{(Southern hemisphere)}$$

$$\phi = \chi + A\sin2\chi + B\sin4\chi + C\sin6\chi + D\sin8\chi$$

in which

$$\chi = \frac{\pi}{2} - z \qquad\qquad\qquad (7.53)$$

$$\tan\frac{z}{2} = \frac{(x_{UPS} - 2,000,000)}{0.994K\sin\lambda} \qquad x_{UPS} \neq 2,000,000$$

$$\tan\frac{z}{2} = -\frac{(y_{UPS} - 2,000,000)}{0.994K\cos\lambda} \qquad y_{UPS} \neq 2,000,000 \text{ (Northern hemisphere)}$$

$$\tan\frac{z}{2} = +\frac{(y_{UPS} - 2,000,000)}{0.994K\cos\lambda} \qquad y_{UPS} \neq 2,000,000 \text{ (Southern hemisphere)}$$

$$K = \frac{2a^2}{b}\left(\frac{1-\varepsilon}{1+\varepsilon}\right)^{\frac{\varepsilon}{2}}$$

$$\varepsilon^2 = \frac{a^2 - b^2}{a^2}$$

The values of A, B, C, and D are found using equation set 7.45.

In the southern hemisphere, replace the computed value of ϕ with $-\phi$.

7.7.3 Finding point scale factor and convergence of the meridian. The point scale factor and convergence of the meridian are

$$k = 0.994 \qquad\qquad \text{at the poles} \qquad\qquad (7.54)$$

$$k = 0.994 \, \frac{r}{N \cos \phi} \qquad\qquad \text{elsewhere}$$

$$\gamma = \lambda \qquad\qquad\qquad \text{(Northern hemisphere)}$$

$$\gamma = -\lambda \qquad\qquad\qquad \text{(Southern hemisphere)}$$

where r and N are given in equation set 7.47.

7.7.4 <u>Accuracy</u>. The accuracy of the UPS equations is based upon the accuracy of their Polar Stereographic counterparts. For the Polar Stereographic projection, the equations for x, y, λ, and q are exact. The equation for ϕ can be iterated until the desired accuracy is obtained.

7.7.5 <u>Area of coverage</u>. The area of coverage for UPS coordinates is defined by zone limits and overlap.

    North zone: The north polar area 84° - 90°

    South zone: The south polar area 80° - 90°

    UTM overlap: 30′ overlap,
      North: 83° 30′
      South: 79° 30′

7.8 <u>The U.S. Military Grid Reference System (MGRS)</u>. The MGRS is an alpha-numeric system, based upon the UTM and UPS map projections, for identifying positions. An MGRS coordinate consists of a zone designation, alphabetic 100,000-meter grid square designator, and numeric coordinates within the 100,000 meter grid square. MGRS coordinates are defined for the primary UTM and UPS areas, but not for the overlap areas.

7.8.1 <u>MGRS coordinates in the UTM area</u>. In the UTM area, MGRS coordinates are based on the ellipsoid, geodetic latitude, UTM zone, easting and northing as follows:

    a. The first two characters of the MGRS coordinate are the two digits of the numeric UTM zone (see Sections 7.4.1.1 and 7.4.1.2.) Leading zeros must be included.

    b. The third character of the MGRS coordinate is a letter representing a band of geodetic latitude. Beginning at 80° south and proceeding northward, 20 bands are lettered C through X, omitting I and O. The bands are all 8° high except band X (72°N to 84°N), which is 12° high.

    c. The fourth and fifth characters of the MGRS coordinate are a pair of letters representing the 100,000-meter grid square. The letter pair can be selected from Figures 7.7 and 7.8 (from DMA TM 8358.1.) First, reduce the UTM northing by multiples of 2,000,000 until the resulting value is in the range of 0 to 1,999,999 meters. Second, reduce the UTM

northing and easting values to the nearest 100,000 meters. Third, locate the corresponding 100,000 meter northing and easting grid lines for the UTM zone number on the figure. The grid square identifier will be immediately above and to the right of the intersection of the easting and northing grid lines on the figure.

Note: The alphabetic method of designating 100,000-meter grid squares has changed over time. Older products may have different grid square designations than those described above. Also, some software was programmed to compute MGRS coordinates based on geographic position and a map of preferred ellipsoids. The boundaries between preferred ellipsoids have changed with time. Therefore, MGRS coordinates computed with such software may not agree with current map products. MGRS computation software should always require input of the ellipsoid. If MGRS coordinates are to be used in conjunction with a map product they should be checked against the map product to verify compatibility.

d. The remainder of the MGRS coordinate consists of the numeric easting and northing values within the 100,000-meter grid square. The left half of the digits is the easting grid value, which is read to the right of the 100,000-meter easting grid line established in step c. above. The right half of the digits is the northing grid value, which is read northward of the 100,000-meter northing grid line established in step c. above. Both the easting and northing grid values are within the range of 0 to 100,000 meters (i.e., zone and grid square designators, only) to 1 meter (i.e., zone and grid designators followed by 10 digits, five for easting grid value and five for northing grid value). Both easting and northing value must have the same resolution and must include leading zeros.

7.8.2 MGRS coordinates in the UPS area. In the UPS area, MGRS coordinates are based on the ellipsoid, geodetic latitude and longitude, easting, and northing as follows:

a. The first character of the MGRS coordinate is
    A in the Southern and Western Hemispheres,
    B in the Southern and Eastern Hemispheres,
    Y in the Northern and Western Hemispheres, and
    Z in the Northern and Eastern Hemispheres as shown in Figures 7.9 and 7.10.

Figure 7.7

USE THIS DIAGRAM FOR:

CLARKE 1866 ELLIPSOID (NORTH AMERICAN 1927 ONLY)
CLARKE 1880 ELLIPSOID
CLARKE 1841 ELLIPSOID

Grid North

Figure 7.7. MGRS 100,000 meter square identification, in the UTM region, for the Bessel 1841, Clarke 1866 Ellipsoid (North American 1927 datum, only), and Clarke 1880 ellipsoids.

ZONES

SET 1	SET 2	SET 3	SET 4	SET 5	SET 6
1 7 13 19 25 31 37 43 49 55	2 8 14 20 26 32 38 44 50 56	3 9 15 21 27 33 39 45 51 57	4 10 16 22 28 34 40 46 52 58	5 11 17 23 29 35 41 47 53 59	6 12 18 24 30 36 42 48 54 60

2,000,000 m

AV BV CV DV EV FV GV HV	JE KE LE ME NE PE QE RE	SV TV UV VV WV XV YV ZV	AE BE CE DE EE FE GE HE	JV KV LV MV NV PV QV RV	SE TE UE VE WE XE YE ZE
AU BU CU DU EU FU GU HU	JD KD LD MD ND PD QD RD	SU TU UU VU WU XU YU ZU	AD BD CD DD ED FD GD HD	JU KU LU MU NU PU QU RU	SD TD UD VD WD XD YD ZD
AT BT CT DT ET FT GT HT	JC KC LC MC NC PC QC RC	ST TT UT VT WT XT YT ZT	AC BC CC DC EC FC GC HC	JT KT LT MT NT PT QT RT	SC TC UC VC WC XC YC ZC
AS BS CS DS ES FS GS HS	JB KB LB MB NB PB QB RB	SS TS US VS WS XS YS ZS	AB BB CB DB EB FB GB HB	JS KS LS MS NS PS QS RS	SB TB UB VB WB XB YB ZB
AR BR CR DR ER FR GR HR	JA KA LA MA NA PA QA RA	SR TR UR VR WR XR YR ZR	AA BA CA DA EA FA GA HA	JR KR LR MR NR PR QR RR	SA TA UA VA WA XA YA ZA

1,500,000 m

AQ BQ CQ DQ EQ FQ GQ HQ	JV KV LV MV NV PV QV RV	SQ TQ UQ VQ WQ XQ YQ ZQ	AV BV CV DV EV FV GV HV	JQ KQ LQ MQ NQ PQ QQ RQ	SV TV UV VV WV XV YV ZV
AP BP CP DP EP FP GP HP	JU KU LU MU NU PU QU RU	SP TP UP VP WP XP YP ZP	AU BU CU DU EU FU GU HU	JP KP LP MP NP PP QP RP	SU TU UU VU WU XU YU ZU
AN BN CN DN EN FN GN HN	JT KT LT MT NT PT QT RT	SN TN UN VN WN XN YN ZN	AT BT CT DT ET FT GT HT	JN KN LN MN NN PN QN RN	ST TT UT VT WT XT YT ZT
AM BM CM DM EM FM GM HM	JS KS LS MS NS PS QS RS	SM TM UM VM WM XM YM ZM	AS BS CS DS ES FS GS HS	JM KM LM MM NM PM QM RM	SS TS US VS WS XS YS ZS
AL BL CL DL EL FL GL HL	JR KR LR MR NR PR QR RR	SL TL UL VL WL XL YL ZL	AR BR CR DR ER FR GR HR	JL KL LL ML NL PL QL RL	SR TR UR VR WR XR YR ZR

1,000,000 m

AK BK CK DK EK FK GK HK	JQ KQ LQ MQ NQ PQ QQ RQ	SK TK UK VK WK XK YK ZK	AQ BQ CQ DQ EQ FQ GQ HQ	JK KK LK MK NK PK QK RK	SQ TQ UQ VQ WQ XQ YQ ZQ
AJ BJ CJ DJ EJ FJ GJ HJ	JP KP LP MP NP PP QP RP	SJ TJ UJ VJ WJ XJ YJ ZJ	AP BP CP DP EP FP GP HP	JJ KJ LJ MJ NJ PJ QJ RJ	SP TP UP VP WP XP YP ZP
AH BH CH DH EH FH GH HH	JN KN LN MN NN PN QN RN	SH TH UH VH WH XH YH ZH	AN BN CN DN EN FN GN HN	JH KH LH MH NH PH QH RH	SN TN UN VN WN XN YN ZN
AG BG CG DG EG FG GG HG	JM KM LM MM NM PM QM RM	SG TG UG VG WG XG YG ZG	AM BM CM DM EM FM GM HM	JG KG LG MG NG PG QG RG	SM TM UM VM WM XM YM ZM
AF BF CF DF EF FF GF HF	JL KL LL ML NL PL QL RL	SF TF UF VF WF XF YF ZF	AL BL CL DL EL FL GL HL	JF KF LF MF NF PF QF RF	SL TL UL VL WL XL YL ZL

500,000 m

AE BE CE DE EE FE GE HE	JK KK LK MK NK PK QK RK	SE TE UE VE WE XE YE ZE	AK BK CK DK EK FK GK HK	JE KE LE ME NE PE QE RE	SK TK UK VK WK XK YK ZK
AD BD CD DD ED FD GD HD	JJ KJ LJ MJ NJ PJ QJ RJ	SD TD UD VD WD XD YD ZD	AJ BJ CJ DJ EJ FJ GJ HJ	JD KD LD MD ND PD QD RD	SJ TJ UJ VJ WJ XJ YJ ZJ
AC BC CC DC EC FC GC HC	JH KH LH MH NH PH QH RH	SC TC UC VC WC XC YC ZC	AH BH CH DH EH FH GH HH	JC KC LC MC NC PC QC RC	SH TH UH VH WH XH YH ZH
AB BB CB DB EB FB GB HB	JG KG LG MG NG PG QG RG	SB TB UB VB WB XB YB ZB	AG BG CG DG EG FG GG HG	JB KB LB MB NB PB QB RB	SG TG UG VG WG XG YG ZG
AA BA CA DA EA FA GA HA	JF KF LF MF NF PF QF RF	SA TA UA VA WA XA YA ZA	AF BF CF DF EF FF GF HF	JA KA LA MA NA PA QA RA	SF TF UF VF WF XF YF ZF

0 m

| m 200,000 300,000 400,000 500,000 600,000 700,000 800,000 m | m 200,000 300,000 400,000 500,000 600,000 700,000 800,000 m | m 200,000 300,000 400,000 500,000 600,000 700,000 800,000 m | m 200,000 300,000 400,000 500,000 600,000 700,000 800,000 m | m 200,000 300,000 400,000 500,000 600,000 700,000 800,000 m | m 200,000 300,000 400,000 500,000 600,000 700,000 800,000 m |

Figure 7.8

USE THIS DIAGRAM FOR:

GEODETIC REFERENCE SYSTEM 1980 ELLIPSOID
INTERNATIONAL ELLIPSOID
WORLD GEODETIC SYSTEM 1984 ELLIPSOID
WORLD GEODETIC SYSTEM 1972 ELLIPSOID
AUSTRALIAN NATIONAL ELLIPSOID
CLARKE 1866 ELLIPSOID (UTM ZONES 47-50 ONLY)
EVEREST ELLIPSOID
SOUTH AMERICAN 1969 ELLIPSOID (GRS 67)

Grid North

Figure 7.8. MGRS 100,000 meter square identification, in the UTM region, for the Australian National, Clarke 1866 (UTM zones 47 – 50, only), Everest, Geodetic Reference System 1980, International, South American 1969 (GRS 67), WGS 1972 and WGS 1984 Ellipsoids.

Use with:
International Ellipsoid
World Geodetic 1972 Ellipsoid
World Geodetic 1984 Ellipsoid

INTERNATIONAL ELLIPSOID

GRID ZONE DESIGNATIONS
180°

WEST 0° EAST

FIGURE 7.9
100,000 METER SQUARE IDENTIFICATIONS
FOR THE
MILITARY GRID REFERENCE SYSTEM

Figure 7.9. MGRS 100,000 meter square identification, in the northern UPS region.

Use with:

International Ellipsoid

Figure 7.10. MGRS 100,000 meter square identification, in the southern UPS region.

Figure 7.11. MGRS grid zone identification.
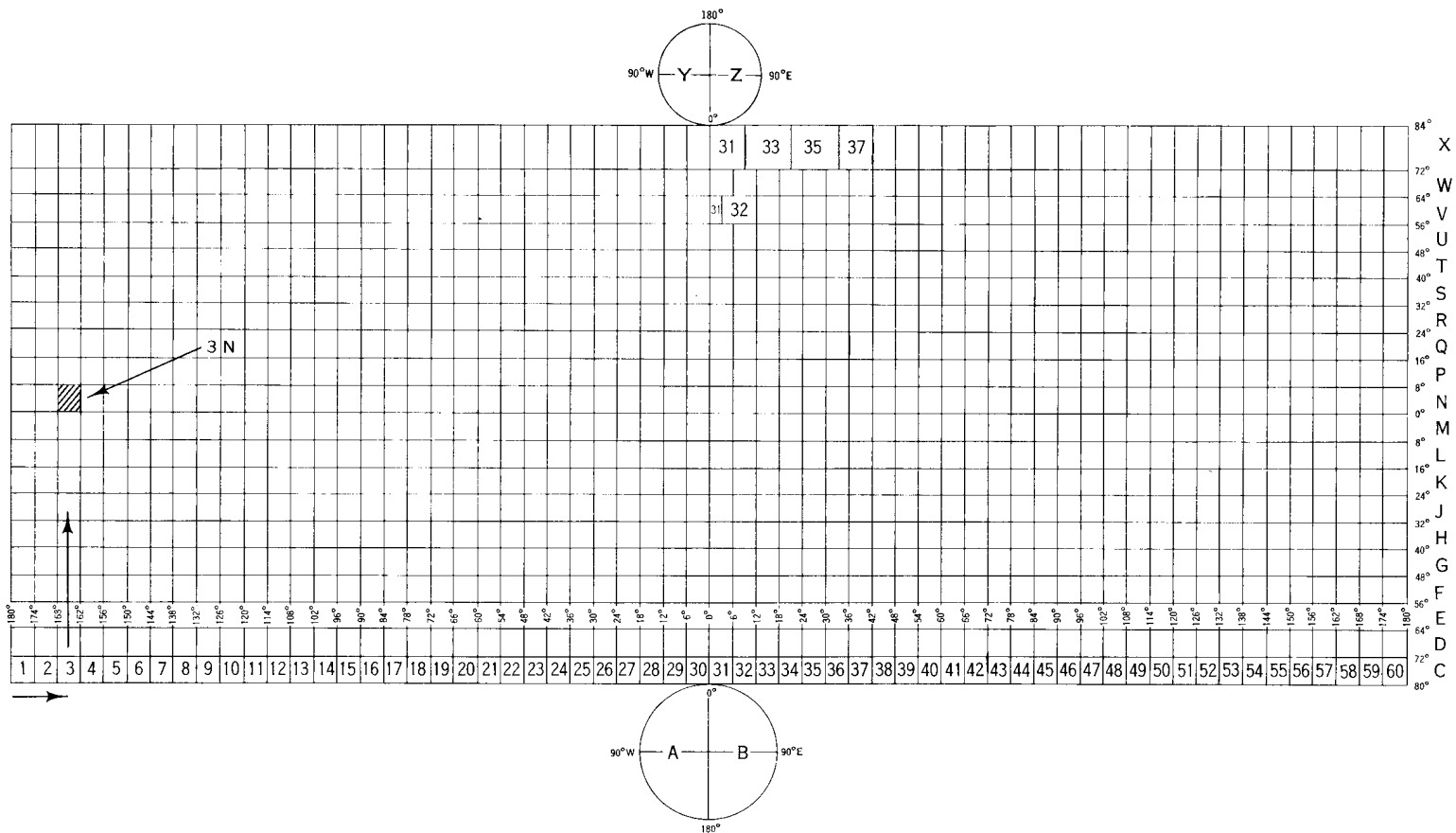
b. The second and third characters of the MGRS coordinate are letters representing a 100,000-meter grid square. The letter pair can be selected from Figures 7.9 or 7.10 (from DMA TM 8358.1.) Reduce the UPS easting and northing values to the nearest 100,000 meters. Locate the corresponding 100,000-meter northing and easting grid lines for the UPS zone number on the figure. The grid square identifier will be immediately above and to the right of the intersection of the easting and northing grid lines on the figure.

c. The method for calculating the remainder of the MGRS coordinate is the same as in paragraph 7.8.1 d.

7.9 The World Geographic Reference System (GEOREF). The GEOREF is an alpha-numeric system for reporting positions for air defense and strategic air operations. It is based upon geodetic coordinates. GEOREF is described in DMA TM 8358.1 (1990).

7.10 Non-standard grids. The specifications and diagrams of non-standard grids, used as the primary or secondary grid on NGA produced maps, are contained in DMA TM 8358.1 (1990).

7.11 Modifications to map projections. Providing equations for transforming between geodetic coordinates and every local map projection is beyond the scope of this handbook; how-ever, many local map projections are minor modifications of the map projections given in this handbook. Typically, these modifications include use of false Eastings (E) and Northings (N) at the projection origin, a non-unity scale factor (k_0) along the principal line of the projection, and units other than meters. Sometimes the local projection origin is offset from the origin used in this handbook. Chapter four of DMA TM 8352.1 (1991) gives parameters for several local pro-jections.

If the local projection is based on one of the projections given in this handbook, convert geodetic latitude (ϕ) and longitude (λ) to coordinates in the local projection by using the following projec-tion:

1. Make sure (ϕ, λ) are on the ellipsoid for the local projection and determine the ellipsoid's parame-ters, a and either f or b.

2. Convert (ϕ, λ) to map projection coordinates (X_p, Y_p) using the appropriate map projection equa-tions given in this handbook. Note that the units for X_p and Y_p are in meters.

3. If the local projection has false coordinates, E, N, at the origin, convert them to units of meters if necessary.

4. The map projection coordinates (X_p, Y_p) and scale factor (k_p) can be converted to local projection coordinates using equation set 7.55 if the origin of the local projection is the same as the origin of that projection as used in this handbook. Those origins are central meridian (λ_0) and equator for Transverse Mercator and Mercator projections, and standard parallel(s) and equator for the Lambert Conformal projection.

$$X = k_0 X_p + E \qquad\qquad\qquad (7.55)$$

$$Y = k_0 Y_p + N$$

$$k = k_0 k_p$$

$$\gamma = \gamma_p$$

where

> E = false X-value (in meters) at the origin
>
> N = false Y-value (in meters) at the origin
>
> k_0 = scale factor along the appropriate line. (Along the equator for the Mercator projection, along the standard parallel(s) for the Lambert Conformal Conic projection, and along the central meridian for the Transverse Mercator projection.)

5. Convert (X, Y) to the units that are locally used. It is the user's responsibility to determine if this procedure is valid. This can be done by comparing computed mapping coordinates to test points where the relationship between (ϕ, λ) and (X, Y) is known, independently.

The above procedure is intended to be used only when the origin of the local map projection coincides with the origin used in this handbook. Some Transverse Mercator projections have origins of (ϕ_0, λ_0) instead of $(0, \lambda_0)$ on the equator. For some of these projections, agreement with test points can be obtained by changing the computation of Y in equation set 7.55 to

$$Y = k_0 Y_{TM} + N - k_0 S_{\phi_0} \qquad\qquad\qquad (7.56)$$

where

> k_0 = scale factor on the central meridian, λ_0
>
> Y_{TM} = Transverse Mercator Y value computed from equation set 7.8
>
> N = false Northing in meters of the origin
>
> S_{ϕ_0} = value computed using equation set 7.9.
>
> (arc length in meters on the ellipsoid from the equator to the latitude ϕ_0.)

8 SELECTION OF TECHNIQUES, SOFTWARE DEVELOPMENT, AND TESTING

8.1 The general conversion process. The general coordinate conversion process can be divided into three basic parts:

1. Conversion between map projection coordinates and geodetic coordinates (see Figure 8.1).

2. Coordinate conversion between geodetic coordinates and Cartesian coordinates within a datum (see Figure 8.2).

3. Coordinate transformation between datums (datum shifts) (see Figure 8.3).

The general conversion process, which includes all methods presented in this handbook, is described graphically in Figure 8.4.

a. Figure 8.1 represents the map projection coordinate (x,y) to the geodetic coordinate (ϕ,λ) conversion described in Section 7. As noted in Section 7.1, map projection coordinates do not include the height information necessary for finding the geodetic height (h) or elevation (H). For conversions from map projection to geodetic coordinates, the user may have to provide this information. Elevations can often be inferred from contour lines on the map or they can be obtained from DTED or direct measurement. Geodetic height may be measured by using GPS or by calculating from an elevation and geoid separation, or for some applications, may be assumed to be 0 (zero).

$$\boxed{\begin{array}{ccc} & A & \\ \mathbf{x,y} \longleftrightarrow & & \phi, \lambda \end{array}}$$
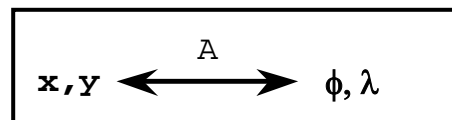
Figure 8.1. Map projection to geodetic coordinate conversion; height information for H and h is not included in the conversion.

where

x, y = map projection coordinates

ϕ = geodetic latitude

λ = geodetic longitude

A = map projection coordinates to/from latitude longitude conversion

b. Figure 8.2 represents coordinate conversions within a datum as described in Section 4.3.4 and 6.5.
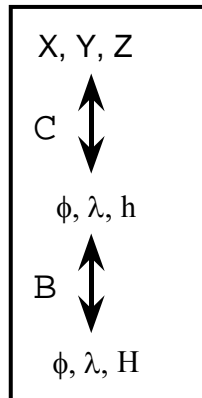
Figure 8.2. Coordinate conversions within a datum.

where

X, Y, Z   = Cartesian coordinates

ϕ, λ, h = geodetic coordinates (latitude, longitude, geodetic height)

H   = elevation

B   = geodetic height to/from elevation conversion

C   = geodetic coordinate to/from Cartesian coordinate conversion

Note that a conversion between Cartesian coordinates (X, Y, Z) and local datum coordinates(ϕ, λ, H) is not a direct conversion and requires going through the geodetic coordinates (ϕ, λ, h).

   c. Figure 8.3 represents the datum shift portion of the general conversion process as described in Section 6. Two attributes of Figure 8.3 are important to note.

      1. Direct local datum to local datum shifts are not performed.

      2. Elevations do not change when changing datums. Exceptions to the second statement are explained in Section 6.7.

   d. In general, a direct local datum to local datum shift should not be done. Instead, the local datum to local datum shift should be done in two steps.

      1. An (old) local datum to WGS 84 shift.

      2. A WGS 84 to (new) local datum shift.
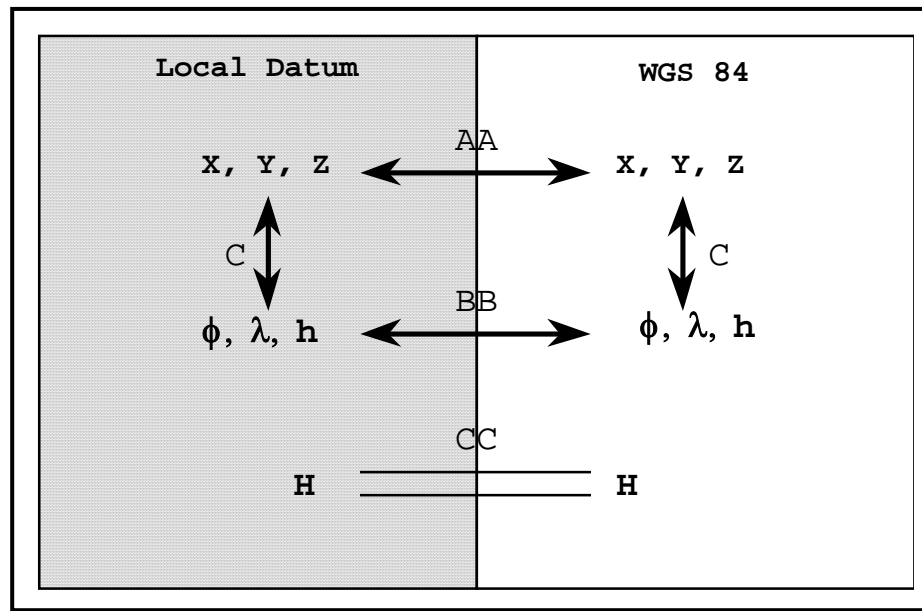
Figure 8.3. Datum shifts; note elevation values (H) do not generally change when changing datums.

where

    X, Y, Z   = Cartesian coordinates

    ϕ, λ, h = geodetic coordinates (latitude, longitude, geodetic height)

    H   = elevation

    C   = geodetic coordinate to/from Cartesian coordinate conversion

    AA   = Cartesian coordinate-based datum shifts

    BB   = geodetic coordinate-based datum shifts

    CC   = elevations are unchanged

e. The general conversion process, which includes all methods presented in this handbook, is described graphically in Figure 8.4. Figure 8.4 is the combination of Figures 8.1 through 8.3,

where

    x, y   = map projection coordinates

    X, Y, Z   = Cartesian coordinates

    ϕ, λ, h = geodetic coordinates (latitude, longitude, geodetic height)
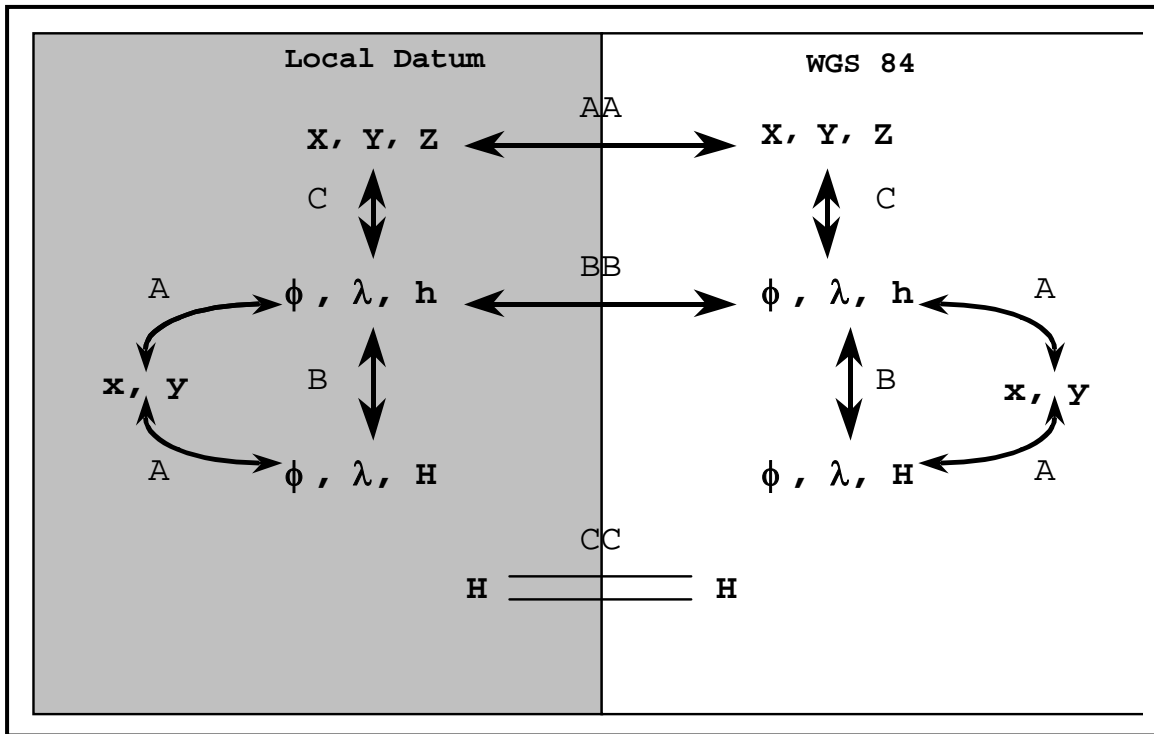
Figure 8.4. The general conversion process.

H = elevation

A = map projection coordinates to/from latitude, longitude conversion

B = geodetic height to/from elevation conversion

C = geodetic coordinate to/from Cartesian coordinate conversion

AA = Cartesian coordinate-based datum shifts

BB = geodetic coordinate-based datum shifts

CC = elevations are unchanged

f.   In Figures 8.1 through 8.4, coordinate conversions are denoted as single letter (A, B, C). They are described in Sections 7, 6.5 and 4.3.4, respectively. Datum shifts are denoted as double letters (AA, BB, CC) and are described in Section 6.

g.   The Cartesian coordinate-based datum shifts (AA) include:
      Seven-parameter geometric transformation   (6.2)
      $\Delta X, \Delta Y, \Delta Z$ direct application   (6.3.1 & 6.3.2)
      Both of these datum shift methods are geometric transformations.

h.   The geodetic coordinate-based datum shifts (BB) include:
      Molodensky shifts (6.3.3)
      $\Delta X, \Delta Y, \Delta Z$ three-step method   (6.3.1.1 & 6.3.2.1)

> Direct WGS 72 to WGS 84 transformation   (6.4.2)

   i. For the purposes of this handbook, the elevation to elevation shift (CC) is the equality function (=); that is, the elevation for the same location on different datums is considered to be the same (H = H). An exception to this rule is the WGS 72-derived elevations to WGS 84 elevations shift. Section 6.7 describes the elevation to elevation shift process.

8.1.1 <u>Application of the general conversion process</u>. The general conversion process described above can be used to solve coordinate conversion/datum shift problems in five steps:

   a. Locate beginning point and ending point of the problem on Figure 8.4.

   b. Determine valid paths between beginning and end points.

   c. Evaluate alternatives and select a path.

   d. Locate the needed equations and parameters in this handbook.

   e. Implement the selected path.

8.1.2 <u>Procedural Examples.</u>

8.1.2.1 <u>Example 8.1</u>. Horizontal geodetic coordinate-based datum shifts [$(\phi, \lambda, h)_{LGS}$ to $(\phi, \lambda)_{WGS\ 84}$]. (Local datum geodetic heights (h_{LGS}) typically are not available. It is not recommended they be transformed to WGS 84).

This handbook has discussed two methods of doing datum shifts. These methods are the three-step method and Molodensky shifts. Either the three-parameter $\Delta X, \Delta Y, \Delta Z$ shift or the seven-parameter shift may be used in the three-step method. This example corresponds to a "BB" shift of Figures 8.3 and 8.4.

   a. <u>Approach 1 - Three-step method using a seven-parameter shift.</u>

      1. Compute $(X, Y, Z)_{LGS}$ from $(\phi, \lambda, h)_{LGS}$ using equation sets 4.4 and 4.5, and the reference ellipsoid parameters from Table A.1.

      2. Compute $(X, Y, Z)_{WGS\ 84}$ from $(X, Y, Z)_{LGS}$ using the seven-parameter shift equation set 6.1 and shift parameters from Table E.1.

      3. Compute $(\phi, \lambda)_{WGS\ 84}$ from $(X, Y, Z)_{WGS\ 84}$ using equation sets 4.6 through 4.10.

   b. <u>Approach 2 - Three-step method using $\Delta X, \Delta Y, \Delta Z$ shifts</u>

      1. Compute $(X, Y, Z)_{LGS}$ from $(\phi, \lambda, h)_{LGS}$ using equation sets 4.4 and 4.5. Reference ellipsoid parameters are found in Table A.1.

2. Compute $(X, Y, Z)_{WGS\ 84}$ from $(X, Y, Z)_{LGS}$ using equation set 6.3, and shift parameters from Table B.1

3. Compute $(\phi, \lambda)_{WGS\ 84}$ from $(X, Y, Z)_{WGS\ 84}$ and equation sets 4.6 through 4.10.

  c. <u>Approach 3 - Using Molodensky shift</u>

1. Compute $\Delta\phi$ and $\Delta\lambda$ from $(\phi, \lambda, h)_{LGS}$ using the Molodensky equation set 6.5, shift parameters from Table B.1, and the local datum's reference ellipsoid parameters given in Table A.1.

2. Compute $(\phi, \lambda)_{WGS\ 84}$ from $(\phi, \lambda)_{LGS}$ using equation set 6.6.

<u>8.1.2.2 Example 8.2</u>. WGS 84 Cartesian coordinate to local latitude, longitude, elevation.

Several paths for converting $(X, Y, Z)_{WGS\ 84}$ to $(\phi_{LGS}, \lambda_{LGS}, H)$ can be traced in Figure 8.4. The steps of three approaches are provided below.

  a. <u>Approach 1 - Using a seven-parameter datum shift</u>

1. Compute $(X, Y, Z)_{LGS}$ from $(X, Y, Z)_{WGS\ 84}$ using the seven-parameter shift equation set 6.2 and parameters from Table E.1.

2. Compute $(\phi, \lambda)_{LGS}$ from $(X, Y, Z)_{LGS}$ using equation sets 4.6 through 4.10 with the parameters of the local ellipsoid.

3. Compute $h_{WGS\ 84}$ from $(X, Y, Z)_{WGS\ 84}$ using equations 4.7 through 4.11 (use equation 4.12 in polar regions).

4. Compute H from $h_{WGS\ 84}$ and $N_{WGS\ 84}$ using equation 5.1, resulting in $\phi_{LGS}, \lambda_{LGS}, H$.

  b. <u>Approach 2 - Using a direct $\Delta X, \Delta Y, \Delta Z$ datum shift</u>

1. Compute $(X, Y, Z)_{LGS}$ from $(X, Y, Z)_{WGS\ 84}$ using the $\Delta X, \Delta Y, \Delta Z$ shift equation set 6.4 and shift parameters from Table B.1.

2. Compute $(\phi, \lambda)_{LGS}$ from $(X, Y, Z)_{LGS}$ using equation sets 4.6 through 4.10 with the parameters of the local ellipsoid.

3. Compute $h_{WGS\ 84}$ from $(X, Y, Z)_{WGS\ 84}$ using equations 4.7 through 4.11. (Use equation 4.12 in polar regions).

4. Compute H from $h_{WGS\ 84}$ and $N_{WGS\ 84}$ using equation 5.1, resulting in $\phi_{LGS}, \lambda_{LGS}, H$.

  c. <u>Approach 3 - Using a Molodensky datum shift</u>

1. Compute $(\phi, \lambda, h)_{WGS\ 84}$ from $(X, Y, Z)_{WGS\ 84}$ using equation sets 4.6 through 4.11. (in polar regions, equation set 4.11 should be replaced by equation set 4.12).

2. Compute $(\phi, \lambda)_{LGS}$ from $(\phi, \lambda)_{WGS\ 84}$ using the Molodensky equation set 6.5, equation set 6.7 and shift parameters from Table B.1.

3. Compute H from $h_{WGS\ 84}$ and $N_{WGS\ 84}$ using equation 5.1, resulting in ϕ_{LGS}, λ_{LGS}, H.

8.1.2.6 Example 8.3. Local latitude, longitude, elevation (ϕ_{LGS}, λ_{LGS}, H) to WGS 84
    latitude, longitude, elevation ($\phi_{WGS\ 84}$, $\lambda_{WGS\ 84}$, H).

Example 8.3 is an extension of example 8.1. Example 8.3 shows the recommended methods for
determining geodetic height used in each of the geometric datum shift methods. The steps of
three alternate approaches are provided as follows.

a. Approach 1 - Using a seven-parameter datum shift

1. Compute $(X, Y, Z)_{LGS}$ from (ϕ_{LGS}, λ_{LGS}, H) using equation sets 4.4 and 4.5.

2. Compute $(X, Y, Z)_{WGS\ 84}$ from $(X, Y, Z)_{LGS}$ using the seven parameter shift equation
    set 6.1 and shift parameters from Table E.1.

3. Compute $(\phi, \lambda)_{WGS\ 84}$ from $(X, Y, Z)_{WGS\ 84}$ using equation sets 4.6 through 4.10.

4. Take H from the input coordinates resulting in $\phi_{WGS\ 84}$, $\lambda_{WGS\ 84}$, H.

b. Approach 2 - Using a ΔX, ΔY, ΔZ datum shift

1. Compute $(X, Y, Z)_{LGS}$ from (ϕ_{LGS}, λ_{LGS}, H) using equation sets 4.4 and 4.5.

2. Compute $(X, Y, Z)_{WGS\ 84}$ from $(X, Y, Z)_{LGS}$ using ΔX, ΔY, ΔZ shift equation set 6.3,
    and shift parameters from Table B.1.

3. Compute $(\phi, \lambda)_{WGS\ 84}$ from $(X, Y, Z)_{WGS\ 84}$ using equation sets 4.6 through 4.10.

4. Take H from the input coordinate resulting in $\phi_{WGS\ 84}$, $\lambda_{WGS\ 84}$, H.

c. Approach 3 - Using a Molodensky datum shift

1. Compute $(\phi, \lambda)_{WGS\ 84}$ from (ϕ_{LGS}, λ_{LGS}, H) using Molodensky equation set 6.5, equa-
    tion set 6.6, and shift parameters from Table B.1.

2. Take H from the input coordinate resulting in $\phi_{WGS\ 84}$, $\lambda_{WGS\ 84}$, H.

8.2 Choosing a datum shift method. Several factors should be considered in selecting a
datum shift method. These factors include: shift parameter availability, timing requirements, ac-
curacy requirements, software availability, hardware limitations, standardization considerations
and potential future requirements. If future requirements are considered as part of the system
development, future product improvements may be minimized. These requirements may include
the need to provide data to other systems and to meet higher accuracy requirements. The discus-
sion below applies to a wide range of Army applications. Special, high accuracy applications are
outside the scope of this handbook (see Section 1.2).

The three-parameter shift is recommended for mapping and charting and most other military application. For Western Europe (EUR-M), and Ordnance Survey of Great Britain 1936 (OGB-M), STANAG 2211 specifies the seven-parameter shifts. The use of MREs is not recommended, although it should be noted that STANAG 2211 specifies the use of MREs for geodetic and geophysical applications where "data are available in geodetic coordinates."

Parameter sets for ΔX, ΔY, ΔZ shifts are found in Appendix B, and seven-parameter geometric shifts are contained in Appendix E. Be sure to use the same method throughout a project, identifying the method chosen in the project documentation so that future users of the data can replicate or backtrace your results.

8.2.1 Parameter availability. For many datums, only mean-value three-parameter ΔX, ΔY, and ΔZ shifts are given in this handbook. Based on system implementation tradeoffs, the system developer can: apply the shifts directly to Cartesian coordinates (see Sections 6.3.1 and 6.3.2); use the three-step method (see Section 6.3.1.1 and 6.3.2.1); or apply shifts to geodetic coordinates using the Molodensky approximations (see Section 6.3.3). For other datums, some combination of seven-parameter geometric shifts, and mean and regional ΔX, ΔY, and ΔZ shifts are available. Select a parameter set from those that meet or come closest to meeting the accuracy requirements of the output product. Base the final selection on standardization requirements and system implementation tradeoffs.

8.2.2 Accuracy. Errors in the shift parameters are the dominant error source in datum shifting. Errors introduced by any of the datum shift algorithms given in this handbook are small compared to the errors in the shift parameters and can generally be neglected; however, take care that accuracy is not degraded by the software and hardware implementation of these algorithms.

a. For the large datums, the regional ΔX, ΔY, ΔZ shifts and seven-parameter shifts are generally considered the most accurate, and mean value ΔX, ΔY, ΔZ shifts have the poorest accuracy. The estimated accuracies of the shift parameters are given in Appendices B and E for each listed datum. The errors in geodetic coordinates may be approximated from the ΔX, ΔY, ΔZ errors listed in Appendix B, using the equation set 6.8.

b. The error estimates and general guidance above must be used judiciously. For a given datum, all types of shift parameters were derived from essentially the same set of control data, and errors were estimated by using a comparison to the same data. For small datums, with poorly distributed control, the better fitting shift type may not be more accurate. The seven-parameter model cannot accurately represent datum rotations and scale changes unless the control is geometrically well distributed across the datum.

c. Regional ΔX, ΔY, ΔZ shifts were developed when there were sufficient data to identify regional variations over the datum as a whole. Regional ΔX, ΔY, ΔZ shifts are more accurate than mean-value ΔX, ΔY, ΔZ shifts.

8.2.3 Implementation issues. System implementation issues may influence the selection of transformation method(s).

a. Computational speed and efficiency are important for some applications, such as real-time navigation. The shift type and method might be selected from those that meet accuracy requirements to minimize computational time. Alternately, shifts might be computed in a background process and applied as fixed values for a period of time or over a given area. The system designer must analyze the impact of such design strategies on system accuracy and performance.

b. For some applications, system computational and data storage capabilities are considerations in selecting and implementing transformations. The system designer must determine that word precision, range, and accuracy of mathematical functions preserve the required accuracy.

c. Code complexity, modularity, and software maintenance are important system design concerns. While the datum shift algorithms themselves are relatively simple, selection logic often is complex.

d. Both future requirements and present needs should be considered. Factors for consideration include: timing requirements, current accuracy requirements, potential future accuracy requirements (such as requirements to provide data for future systems), effects of improved databases, and the effects of improvements in other inputs. In other words, the savings in future planned product improvements should be evaluated against today's costs.

e. It is possible that a software developer may choose to use a transformation technique that is more accurate than is required. In such a case, the results are not necessarily superior to those arising from a less accurate, but acceptable, technique. The data are simply degraded less by the more accurate method; the transformed data can be no better than the input data. For example, techniques that achieve geodetic accuracy must be used when geodetic accuracy is needed. Such techniques can be used with a 1:250,000-scale product, but the transformed data still have the accuracy of a 1:250,000 product.

f. Only a limited number of digits can be represented in a storage location of a computer, regardless of where the decimal point occurs. This number can vary from one computer system to another. Many computer systems store a single precision representation of a number containing seven digits. An attempt to store more than seven digits can result in a round-off error. This has unfortunate implications in mapping, charting and geodesy since coordinates frequently contain more than seven required digits. For example, the UTM northing of many locations on the earth is greater than 1,000,000. Consequently, the northing of many locations cannot be represented with sub-meter accuracy in single precision form.

g. Algorithms to calculate mathematical functions may be implemented in hardware (e.g., math co-processor) or software, depending on available hardware and compiler options. Each implementation has an associated accuracy related to computation precision.

While use of single precision may not introduce errors greater than the uncertainties in datum shift parameters, it could still have detrimental consequences. For example, when shifting a survey network to another datum, the relative accuracy between network points should be preserved in the new datum. Rounding and other errors associated with single precision may seriously compromise relative accuracy.

There is a second issue regarding accuracy of single precision storage. Numbers sufficiently large cannot be represented in single precision form even if they have seven or less significant digits. For example, many computers cannot represent ten to the 39th power in single precision form. While no mapping parameter exceeds this limit, take care that successive products of mapping parameters do not exceed the magnitude limit of single precision representation.

As a general rule, use double precision floating point representations when developing software based on transformations found in this handbook. Perform careful analysis of present and future needs, together with anticipated hardware and software platforms, before using single precision.

8.2.3.1 <u>Preferred shift methods</u>. Preferred shift methods and shift parameters may change as additional information becomes available. Shift parameters and identification information should be treated as data apart from the algorithm code so that they may be updated without modifying and recompiling the code. Some examples of implementation tradeoffs are:

    a. If Cartesian coordinates are already available, ΔX, ΔY, ΔZ shifts can be applied directly (see Sections 6.3.1 and 6.3.2) rather than through the Molodensky approximations (see Section 6.3.3).

    b. If the program contains algorithms for converting between Cartesian and geographic coordinates (see Section 4.3.4), the three-step method (see Sections 6.3.1.1 and 6.3.2.1) can be used instead of Molodensky approximations (see Section 6.3.3) to apply ΔX, ΔY, ΔZ shifts.

    c. Both the Molodensky approximations (Section 6.3.3) and Cartesian-to-geographic conversion equations (Section 4.3.4.2) become singular at the poles. Selection of one technique or the other will reduce the logic needed to guard against singularity. An alternate equation to compute geodetic height from Cartesian coordinates in polar regions is given as equation 4.12.

8.2.3.2 <u>Software testing</u>. When developing software based on methods discussed in this handbook, it is advisable to check the software against test points. Test points can be found in Section 8.5.

    a. Issues relating to software development and system performance might force a software developer to use a special-purpose transformation technique that is not mentioned in this

handbook. The results of the special-purpose software should be compared to those of the techniques in this handbook to ensure that the differences are acceptable.

b. Another useful debugging aid is **circular testing.** This is done by using the output from a transformation as the input for the inverse function; in other words trying to verify the original numbers are returned within acceptable accuracy. Since the code for a transformation and its inverse might contain mistakes that cancel each other, circular testing cannot prove that code is correct. At best, it can demonstrate that code is faulty.

c. The following is a list of points that might prove troublesome to coordinate transformation software. Test the code at and near these points, if it is appropriate:
Poles
Polar regions
Discontinuity in λ (transition between 0, 360 degrees or -180, 180 degrees)
Origin expressed in (ϕ, λ)
$\phi = 0$
$\lambda = 0$
Both hemispheres in ϕ (North and South)
Both hemispheres in λ (East and West)
$x = 0$
$y = 0$
All four x-y quadrants
Discontinuities in x and y at zone boundaries, equator, etc.

8.3 Error analysis. Error analysis is an important part of the system design process. Error analysis will estimate the accuracy of the system's output products. This is a prediction of how well the system will meet its accuracy requirements. The system analyst can use error analysis to perform system implementation studies. The major steps in error analysis are:

a. Develop the system model

b. Determine the estimated errors for each source in the model

c. Reformulate the estimated errors to a common description, and

d. Propagate the errors through the model.

Each of these topics is discussed below. An example is given in 8.3.5.

8.3.1 System model. The system model simply is a description of the data used in the process and all the steps (processes) used to process the data. The data include the raw input data and parameters, like datum shift parameters, used in the processing of the data. Both hardware-driven and software-driven processes should be included in the model. For example, errors in digitizing and plotting are primarily hardware related; coordinate conversion and datum shifting

are primarily software processes. For the datum shifting and coordinate conversion, the system model will probably be a simple, serial process. That is, the output of one process will be the input for the next, with no feedback to prior processes. For trade-off studies, alternate system models can be developed to evaluate accuracy versus implementation.

8.3.2 Error estimates. Once the system error model has been described, error estimates must be assigned to each process and data source in the model. When this is done, the analyst will quickly determine which error sources can be neglected and which must be propagated through the model.

   a. Estimates of the expected errors in the input data may be provided with the data or in product specifications. Also, the original source or intended use of the data may provide an indication of its accuracy (e.g., digital data derived from a cartographic source will have an accuracy no better than the cartographic source).

   b. Without proper guidance, it is often difficult to estimate the error in the data. This is especially true for cartographically derived elevations, because map accuracy standards for elevation allow for error in both the horizontal location and the vertical direction. Therefore, the allowable elevation error is dependent on the topography and may vary within a map sheet.

   c. In addition to providing datum shift parameters, Appendices B and E give the estimated errors in the parameters for ΔX, ΔY, ΔZ; and seven-parameter shifts, respectively. (Note: Correlations between parameters, which may be significant, were not available. Therefore, they are not given in the appendices.)

   d. The accuracy and behavior of the equations presented in this handbook are well defined. Table 8.1 identifies the location of accuracy guidance for the coordinate conversion, datum shift, and map projection equations. Additional references describing equation accuracy and behavior are noted in the text and listed in Section 2.

   e. The errors introduced by system implementation factors such as computer word size, accuracy, and precision of mathematical functions, truncation of terms, etc., must be quantified.

   f. The errors introduced by hardware processes must be determined from equipment specifications, analysis, and/or testing.

Table 8.1. Accuracy guidance for equations.

Subject	Accuracy guidance section(s)
Coordinate Conversion Equations	
Cartesian/geodetic conversion	4.3.4
Height conversion	4.4.3
Datum shift Equations	
Seven-parameter geometric transformation	6.2.4
$\Delta X, \Delta Y, \Delta Z$ shifts	6.3.5
Direct WGS 72 to WGS 84 transformation	6.4.2.2
Vertical datum shifts	6.7, 6.7.1, 6.7.2
Map projection Equations	
Mercator projection	7.2.3
Transverse Mercator projection	7.3.3
Universal Transverse Mercator (UTM) grid	7.4.4
Lambert Conformal Conic projection	7.5.4
Polar Stereographic	7.6.2
Universal Polar Stereographic	7.7.4
Shift Parameter Accuracy values	
$\Delta X, \Delta Y, \Delta Z$ shifts	Appendix B
Seven-parameter geometric transformation	Appendix E
Direct WGS 72 to WGS 84 transformation	Section 6.4.2.2

8.3.2.1 Error types. Depending on the source, error estimates may be described in several ways. Error descriptions may have a bias component and a random (noise) component. A set of error estimates describing the accuracy in multiple dimensions, for example, $\sigma_{\Delta X}$, $\sigma_{\Delta Y}$, $\sigma_{\Delta Z}$, may have correlations between the parameters. Generally, with MC&G products, the error descriptions do not differentiate between biased, correlated, and independent random errors; the assumption is made that the error estimates are unbiased, independent, and random. The resultant error analysis will be less accurate than if the true statistical natures of the error sources were known.

8.3.3 Reformulation. Once the error sources have been quantified, they must be converted to a common frame of reference. The form used for the system output error estimate is often a convenient frame of reference.

   a. The reference for the error estimates must be established (i.e., the ellipsoid or the map sheet). It may be necessary to scale from one reference to another.

   b. The error estimates should be converted to the same units.

c. The datum shift parameter error estimates, $\sigma_{\Delta X}$, $\sigma_{\Delta Y}$, $\sigma_{\Delta Z}$, given in Appendix B, can be re-formulated to $\sigma_{\Delta\phi}$, $\sigma_{\Delta\lambda}$, $\sigma_{\Delta H}$ using equation set 6.8.

d. Error estimates typically are linear, circular, or spherical statistical values with associated probability levels. Error estimates should be converted to the same type of statistic at the same probability level. Two independent linear standard deviations (68.27% probability), σ_x and σ_y, can be converted to a circular standard deviation (39.35% probability), using

$$\sigma_c \approx \frac{(\sigma_x + \sigma_y)}{2} \qquad (8.1)$$

if the values of σ_x and σ_y are reasonably close. Factors to convert between different levels of probability are given in Table 8.2.

8.3.4 Error propagation. Assuming the error estimates are statistically independent and have been converted to the same units and statistical type, they may be propagated through a simple, serial system model using the Root-Sum-Square (RSS) technique.

$$RSS = \sqrt{(N_1)^2 + (N_2)^2 + \ldots + (N_n)^2} \qquad (8.2)$$

If error estimates have known biases or correlations, then a more sophisticated error propagation technique should be used.

Table 8.2. Probability level conversion factors

Notation				
Error Form	Standard Error (One Sigma)	Probable Error	Map Accuracy Standard	Near Certainty Error
Linear	σ (68.27%)	PE (50%)	MAS (90%)	3σ (99.73%)
Circular	σ_c (39.35%)	CEP (50%)	CMAS (90%)	$3.5\sigma_c$ (99.78%)
Spherical	σ_s (19.9%)	SEP (50%)	**SAS (90%)	$4\sigma_s$ (99.89%)
**SAS = Spherical Accuracy Standard				
Linear Error (One dimensional)				
To From	50%	68.27%	90%	99.73%
50%	1.0000	1.4826	2.4387	4.4475
68.27	0.6745	1.0000	1.6449	3.0000
90	0.4101	0.6080	1.0000	1.8239
99.73	0.2248	0.3333	0.5483	1.0000

Circular Error (Two dimensional)				
To From	39.35%	50%	90%	99.78%
39.35%	1.0000	1.1774	2.1460	3.5000
50	0.8493	1.0000	1.8227	2.9726
90	0.4660	0.5486	1.0000	1.6309
99.78	0.2857	0.3364	0.6131	1.0000
Spherical Error (Three Dimensional)				
To From	19.9%	50%	90%	99.89%
19.9%	1.000	1.538	2.500	4.000
50	0.650	1.000	1.625	2.600
90	0.400	0.615	1.000	1.600
99.89	0.250	0.385	0.625	1.000
(Note: Errors are assumed to be random and uncorrelated.)				

8.3.5 Example. The commander wants a printed product showing the transportation routes near Seoul, Korea with WGS 84, UTM coordinates, at a scale of 1:50,000. The data source is a Class A, 1:50,000-scale topographic map on the Tokyo datum. The product will be created by digitizing the transportation routes on the map sheet, shifting the coordinates from the Tokyo datum to WGS 84 using the three-step method, and plotting the shifted data.

   a. The system model is shown in Figure 8.5. In this model, it is assumed that computer-related issues will not degrade coordinate conversion and datum shift calculations. Also, it is assumed that the representation of the digitized data will not change through the rest of the process. Often this assumption is not valid because digital data representation must be changed to a predetermined format. Such data modifications can introduce error.

   b. The error estimates for each of the elements in the model are

Element	Error
Map accuracy (horizontal)	0.02" CMAS(90%)
Digitizer	±0.005″
UTM to Geodetic conversion	<1 cm
Geodetic to XYZ conversion	0.0
Datum shift equations	0.0
Datum shift parameters:	
$\sigma_{\Delta X}$	5 m
$\sigma_{\Delta Y}$	3 m
$\sigma_{\Delta Z}$	3 m
XYZ to Geodetic conversion	0.1 m
Geodetic to UTM conversion	<1 cm
Plotter	±00.01″

Only the map, digitizer, datum shift parameter, and plotter errors are significant and will be considered in the rest of the analysis.

c.   The error estimates must be reformulated to the same units and statistical basis. Since the commander wants to know the estimated error on the ground, we will convert to circular error, in meters, on the ellipsoid, at 50 percent probability (CEP).

   1.   Map error must be scaled from the map to the ellipsoid and changed from CMAS to CEP.

   Map Error   = 0.02″ CMAS x 0.0254 m/in x 50,000

                        = 25.4 m CMAS x 0.5486

                        = 13.9 m CEP

   2.   It is assumed that the digitizer error is a near certainty, circular error specification that must be scaled from the map to the ellipsoid and converted to CEP.

   Digitizer error   = 0.005″ max x 0.0254 m/in x 50,000

                        = 6.4 m max x 0.3364

                        = 2.1 m CEP

   3.   The datum shift parameter error estimates, $\sigma_{\Delta X}$, $\sigma_{\Delta Y}$, $\sigma_{\Delta Z}$, must be converted to $\sigma_{\Delta \phi}$ and $\sigma_{\Delta \lambda}$, then to CEP.

   Using the above values for $\sigma_{\Delta X}$, $\sigma_{\Delta Y}$, and $\sigma_{\Delta Z}$, $\phi = 37.5°$ and $\lambda = 127°$, equation set 6.8 yields $\sigma_{\Delta \phi} = 3.34$ m and $\sigma_{\Delta \lambda} = 4.38$ m.

$$CEP = 1.1774\ \sigma_c \approx 1.1774\ \frac{\sigma_{\Delta \phi} + \sigma_{\Delta \lambda}}{2} = 4.5\ m$$

   4.   It is assumed that the plotter error is a near certainty, circular error (99.78% probability) specification that must be scaled from the map to the ellipsoid and converted to CEP.

   Plotter error   = 0.01″ max x 0.0254 m/in x 50,000

                        = 12.7 m max x 0.3364

                        = 4.3 m CEP

   5.   The RSS error for the complete system model is

$$RSS = \sqrt{(13.9)^2 + (2.1)^2 + (4.5)^2 + (4.3)^2} = 15.4\ m\ CEP$$

d.   From the above analysis, it is evident that the error in the source data is the prime contributor to error in the output product for this application. For other types of applications, such as surveying, this may not be the case.
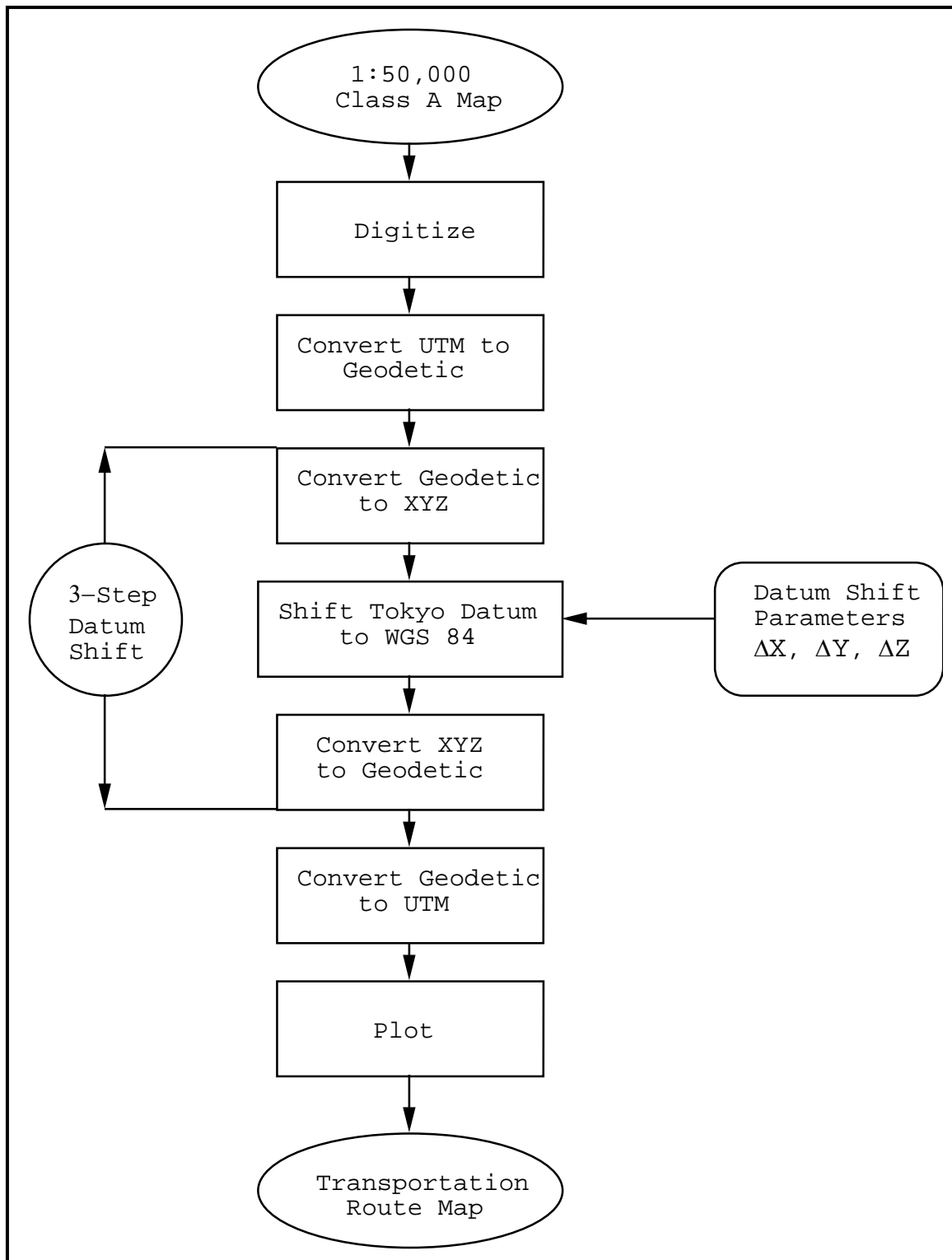
Figure 8.5. System model

8.4 Numerical Examples.

8.4.1 Example 8.4. Convert WGS 84 geodetic coordinates (ϕ, λ, h)$_{\text{WGS 84}}$ to Universal
    Transverse Mercator coordinates (x, y)$_{\text{UTM}}$ in the NAD 27 reference system using a
    three-step ΔX, ΔY, ΔZ datum shift.

This example converts WGS 84 geodetic coordinates to Universal Transverse Mercator projec-
tion coordinates in two steps. First, the WGS 84 geodetic coordinates are converted to NAD 27
geodetic coordinates using the three-step ΔX, ΔY, ΔZ datum shift. Geodetic height is needed for
this conversion. Next, the NAD 27 geodetic latitude and longitude are converted to UTM coor-
dinates. Geodetic height is not needed for this step.

Beginning WGS 84 coordinates:

   ϕ = 42° 56′ 52.163″N = 0.749580918883376 radians

   λ = 288° 22′ 24.326″E = 5.0330657221104 radians

   h = 203.380 m

Step 1: Convert WGS 84 geodetic coordinates to NAD 27 geodetic coordinates using the three-
    step ΔX, ΔY, ΔZ datum shift.

   1.1 Convert (ϕ, λ, h)$_{\text{WGS 84}}$ to (X, Y, Z)$_{\text{WGS 84}}$ using equation sets 4.4 and 4.5.

      a = 6378137

      f = 0.00335281066474

      $\varepsilon^2 = 2f - f^2 = 0.00669437999013$

      b = a(1 − f) = 6356752.3142

      $R_N = \dfrac{a}{\sqrt{1 - \varepsilon^2 \sin^2 \phi}} = 6388070.57383009$

      $X_{\text{WGS 84}} = (R_N + h) \cos \phi \cos \lambda = 1473933.5413$

      $Y_{\text{WGS 84}} = (R_N + h) \cos \phi \sin \lambda = -4437679.0666$

      $Z_{\text{WGS 84}} = \left(\dfrac{b^2}{a^2} R_N + h\right) \sin \phi = 4323399.2717$

1.2   Convert $(X, Y, Z)_{WGS\ 84}$ to $(X, Y, Z)_{NAD\ 27}$ using equation set 6.4 and ΔX, ΔY, ΔZ datum shift values from Table B.1 NAS-C.

$$X_{NAD\ 27} = X_{WGS\ 84} - \Delta X = 1473933.5413 - (-8.0) = 1473941.5413\ m$$

$$Y_{NAD\ 27} = Y_{WGS\ 84} - \Delta Y = -4437679.0666 - 160.0 = -4437839.0666\ m$$

$$Z_{NAD\ 27} = Z_{WGS\ 84} - \Delta Z = 4323399.2717 - 176 = 4323223.2717\ m$$

1.3 Convert $(X, Y, Z)_{NAD\ 27}$ to $(\phi,\ \lambda)_{NAD\ 27}$ following equations given in Section 4.3.4.2.

$$\lambda_{NAD\ 27} = \arctan\left(\frac{Y_{NAD\ 27}}{X_{NAD\ 27}}\right) = 5.033056560575\ radians$$

Calculating ϕ is an iterative process.

1.3.1   Compute an initial approximation to a variable β, using equation 4.7. Parameters a and b for the Clarke 1866 ellipsoid are found in Table A.

$$\beta_0 = \arctan\left(\frac{a\ Z}{b\ \sqrt{X^2 + Y^2}}\right) = 0.74788820090716142\ radians$$

1.3.2   Substitute β_0 for β in equation 4.8 to get an approximation for ϕ.

$$\phi = \arctan\left(\frac{Z + \varepsilon'^2\ b\ \sin^3 \beta}{\sqrt{X^2 + Y^2} - a\ \varepsilon^2\ \cos^3 \beta}\right) = 0.74958142853674489\ radians$$

$$\varepsilon^2 = 2f - f^2 = 0.0067686579973$$

and

$$\varepsilon'^2 = \frac{\varepsilon^2}{1 - \varepsilon^2} = 0.0068147849459$$

For most applications, this first iteration is sufficient.

Step 2: Convert the NAD 27 geodetic latitude and longitude to Universal Transverse Mercator projection coordinates.

2.1 Find the UTM zone z and the central meridian λ_0 from equations 7.22 and 7.23.

$$z = greatest\ integer \leq \left(\frac{180\ \lambda}{6\pi} - 29\right)$$

$$z = 19$$

$$\lambda_0 = (6z + 177)\frac{\pi}{180} = 5.078908123303\ radians$$

2.2   Calculate Transverse Mercator projection coordinates (X_{TM}, Y_{TM}) using equations 7.8 and 7.9.

   2.2.1   Calculate terms in equation set 7.9 using ε^2 and ε'^2 as calculated in Step 1.3.2.

$$A_0 = 1 - \frac{1}{4}\,\varepsilon^2 - \frac{3}{64}\,\varepsilon^4 - \frac{5}{256}\,\varepsilon^6 - \frac{175}{16384}\,\varepsilon^8 = 0.998305681856$$

$$A_2 = \frac{3}{8}\left(\varepsilon^2 + \frac{1}{4}\,\varepsilon^4 + \frac{15}{128}\,\varepsilon^6 - \frac{455}{4096}\,\varepsilon^8\right) = 0.2542555420215 \times 10^{-2}$$

$$A_4 = \frac{15}{256}\left(\varepsilon^4 + \frac{3}{4}\,\varepsilon^6 - \frac{77}{128}\,\varepsilon^8\right) = 0.2698010542695 \times 10^{-5}$$

$$A_6 = \frac{35}{3072}\left(\varepsilon^6 - \frac{41}{32}\,\varepsilon^8\right) = 0.3502448582027 \times 10^{-8}$$

$$A_8 = -\frac{315}{131072}\,\varepsilon^8 = -0.5044416191579 \times 10^{-11}$$

$$S_\phi = a\,[A_0\,\phi - A_2\,\sin 2\phi + A_4\,\sin 4\phi - A_6\,\sin 6\phi + A_8\,\sin 8\phi]$$

$$S_\phi = 4756711.680282$$

$$N = R_N = \frac{a}{\sqrt{1 - \varepsilon^2\,\sin^2\phi}} = 6388250.562973$$

Note that $N = R_N$ differs from R_N calculated in Step 1.1, as Step 2 is in the NAD 27 reference frame and Step 1.1 is in the WGS 84 reference frame.

$$\eta = \varepsilon'\,\cos\phi = 0.6042571576496 \times 10^{-1}$$

$$t = \tan\phi = \tan(0.74958142853674489) = 0.9308149267480$$

$$\Lambda = \lambda - \lambda_0 = -0.04585156272818 \text{ radians}$$

   2.2.2   Calculate the Transverse Mercator projection coordinates (x_{TM}, y_{TM}).

$$x_{TM} = N\,\Lambda\,\cos\phi + \frac{N\,\Lambda^3\,\cos^3\phi}{6}\,(1 - t^2 + \eta^2)$$
$$+ \frac{N\,\Lambda^5\cos^5\phi}{120}\,(5 - 18t^2 + t^4 + 14\,\eta^2 - 58\,t^2\,\eta^2)$$

$$y_{TM} = S_\phi + \frac{N \Lambda^2}{2} \sin \phi \cos \phi + \frac{N \Lambda^4}{24} \sin \phi \cos^3 \phi (5 - t^2 + 9 \eta^2 + 4 \eta^4)$$

$$+ \frac{N \Lambda^6}{720} \sin \phi \cos^5 \phi (61 - 58 t^2 + t^4 + 270 \eta^2 - 330 t^2 \eta^2)$$

$$x_{TM} = -214408.9715411$$

$$y_{TM} = 4760061.989246$$

2.3 Convert the Transverse Mercator projection coordinates (x_{TM}, y_{TM}) to Universal Transverse Mercator projection coordinates (x_{UTM}, y_{UTM}) using equation set 7.24 (northern hemisphere).

$$x_{UTM} = 0.9996 \ x_{TM} + 500,000 = 0.9996 \ (-214408.9715411) + 500,000$$

$$y_{UTM} = 0.9996 \ y_{TM} = 0.9996 \ (4760061.989246)$$

The resulting Universal Transverse Mercator coordinates (x_{UTM}, y_{UTM}) in the NAD 27 reference system are:

$$x_{UTM} = 285676.792$$

$$y_{UTM} = 4758157.964$$

8.4.2 Example 8.5. Convert WGS 84 geodetic coordinates (ϕ, λ, h) to geodetic coordinates (ϕ, λ, h) in the NAD 27 reference system using the standard Molodensky method.

This example converts WGS 84 geodetic coordinates to NAD 27 coordinates using the standard Molodensky method.

Beginning WGS 84 coordinates:

$\phi = 42° \ 56' \ 52.163'' \ N = 0.749580918883376$ radians

$\lambda = 108° \ 22' \ 24.326'' \ W = 1.891473118$ radians

$h = 203.380$

Step 1:   Compute Δa, Δf, W^2, M, N, ε^2, and ε'^2 using equation set 6.5. Obtain $a_{WGS \ 84}$, $a_{CLARKE \ 1866}$, $f_{WGS \ 84}$, and $f_{CLARKE \ 1866}$ from Table A.1. Note NAD 27 uses the Clarke 1866 ellipsoid.

$\Delta a = a_{WGS \ 84} - a_{CLARKE \ 1866} = -69.4$

$\Delta f = f_{WGS \ 84} - f_{CLARKE \ 1866} = 3.726463863 \ x \ 10^{-5}$

$$\varepsilon^2 = 2f - f^2 = 0.006694379990$$

$$\varepsilon'^2 = \frac{\varepsilon^2}{1 - \varepsilon^2} = 0.006739496742$$

$$W = \sqrt{1 - \varepsilon^2 \sin^2 \phi} = 0.9984449805$$

$$N = R_N = \frac{a}{W} = 6388070.574$$

$$M = \frac{a(1 - \varepsilon^2)}{W^3} = 6365086.681$$

Step 2: Compute $\Delta\phi$ and $\Delta\lambda$, and Δh using equation set 6.5, and ΔX, ΔY, and ΔZ datum shifts from Appendix B for datum code NAS-A. In this example, the signs of ΔX, ΔY, and ΔZ are reversed from those of Appendix B due to the direction of the datum shift (WGS 84 to local coordinates).

$$\Delta\phi = [-\Delta x \sin \phi \cos \lambda - \Delta y \sin \phi \sin \lambda + \Delta z \cos \phi + \frac{\varepsilon^2 \sin \phi \cos \phi}{W} \Delta a$$

$$+ \sin \phi \cos \phi \, (2 \, N + \varepsilon'^2 \, M \sin^2 \phi)(1 - f) \, \Delta f]/(M + h)$$

$$\Delta\phi = 0.131''$$

$$\Delta\lambda = \frac{-\Delta x \sin \lambda + \Delta y \cos \lambda}{(N + h) \cos \phi} = 2.615''$$

$$\Delta h = \Delta x \cos \phi \cos \lambda + \Delta y \cos \phi \sin \lambda + \Delta z \sin \phi - W \Delta a + \frac{a \, (1 - f)}{W} \Delta f \sin^2 \phi$$

$$\Delta h = 28.65 \text{ m}$$

Step 3: Compute the NAD 27 geodetic coordinates (ϕ, λ, h) from equation set 6.7 that results in the following values:

$$\phi_{NAD \, 27} = \phi_{WGS \, 84} + \Delta\phi = 42° \, 56' \, 52.294'' \, N$$

$$\lambda_{NAD \, 27} = \lambda_{WGS \, 84} + \Delta\lambda = 108° \, 22' \, 21.711'' \, W$$

$$h_{NAD \, 27} = H_{WGS \, 84} + \Delta H = 232.03 \text{ m}$$

8.4.3 Example 8.6. Convert WGS 84 geodetic coordinates $(\phi, \lambda)_{WGS \, 84}$ to Mercator projection coordinates (x, y).

This example converts WGS 84 geodetic coordinates to Mercator projection coordinates within the WGS 84 reference system. Geodetic height (h) is not used in this conversion. The Mercator projection central meridian λ_0 is 288° = 5.026548246 radians

Beginning WGS 84 coordinates are

$$\phi_{WGS\ 84} = 42° 56' 52.163'' N = 0.7495809189 \text{ radians}$$

$$\lambda_{WGS\ 84} = 288° 22' 24.326'' E = 5.033057346 \text{ radians}$$

Step 1: Use equation 7.1 and WGS 84 datum parameters a and f from Table A.1 to calculate x.

$$x = a\ (\lambda - \lambda_0) = 6378137(5.033057346 - 5.026548246) = 41569.357$$

Step 2: Calculate ε from f using equation 4.9, then use equation 7.2 to calculate y.

$$\varepsilon = \sqrt{2f - f^2} = 0.0818191908426$$

$$y = a \ln\left[\tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\left(\frac{1 - \varepsilon \sin \phi}{1 + \varepsilon \sin \phi}\right)^{\frac{\varepsilon}{2}} \right] = 5274911.868$$

The resulting Mercator projection coordinates (x, y) are

$$x = 41569.357$$

$$y = 5274911.868$$

8.4.4 Example 8.7. Convert Mercator projection coordinates (x, y) in the WGS 84 reference system to WGS 84 geodetic latitude and longitude (ϕ, λ).

This example converts WGS 84 Mercator coordinates to WGS 84 latitude and longitude.

Beginning Mercator coordinates and central meridian (λ_0):

$$x = 41569.3572$$

$$y = 5274911.8684$$

$$\lambda_0 = 288°E = 5.0265482457 \text{ radians}$$

Step 1: Compute the longitude (λ) using equation 7.4 where a is the semi-major axis of the WGS 84 ellipsoid listed in Table A.1.

$$\lambda = \frac{x}{a} + \lambda_0 = 5.0330657221 \text{ radians} \quad = 288° 22' 24.326''E = 71° 37' 35.674''W$$

Step 2: Compute an intermediate value t from equation 7.5.

$$t = e^{\left(-\frac{y}{a}\right)} = e^{\left(-\frac{5274911.8684}{6378137}\right)} = 0.43734619376064$$

Step 3: Compute an initial value of the geodetic latitude (ϕ_0) from equation 7.6.

$$\phi_0 = \frac{\pi}{2} - 2 \arctan(t) = 0.746233651 \text{ radians}$$

Step 4: Iterate equation 7.7 until the updated value of ϕ is sufficiently close to the previous value of ϕ.

$$\varepsilon^2 = 2f - f^2 = 0.00669437999013$$

$$\varepsilon = 0.0818191908426$$

$$\phi_{n+1} = \frac{\pi}{2} - 2 \arctan\left[t \left(\frac{1 - \varepsilon \sin \phi_n}{1 + \varepsilon \sin \phi_n}\right)^{\frac{\varepsilon}{2}} \right]$$

$$\phi_1 = 0.7495688569418636$$

The geodetic latitude will be given by the last value of ϕ calculated.

$$\text{Final } \phi = 0.7495809188833704 \text{ radians} = 42° 56' 52.163''N$$

The resulting WGS 84 geodetic latitude and longitude (ϕ, λ) are:

$$\phi_{\text{WGS 84}} = 42° 56' 52.163'' N$$

$$\lambda_{\text{WGS 84}} = 288° 22' 24.326'' E = 71° 37' 35.674'' W$$

8.4.5 Example 8.8. Convert Universal Transverse Mercator coordinates (x_{UTM}, y_{UTM}) in the NAD 27 reference system to NAD 27 geodetic latitude and longitude(ϕ, λ).

This example converts NAD 27 UTM coordinates to NAD 27 geodetic latitude and longitude. The UTM coordinates are converted to Transverse Mercator projection coordinates. The Transverse Mercator projection coordinates are then converted to NAD 27 latitude and longitude.

Beginning coordinates are

$$x_{UTM} = 285{,}677.332$$

$$y_{UTM} = 4{,}758{,}154.856$$

$$\text{Zone} = 19$$

Step 1: Convert Universal Transverse Mercator coordinates (x_{UTM}, y_{UTM}) to Transverse Mercator coordinates (x_{TM}, y_{TM}) using equation set 7.25.

$$x_{TM} = \frac{1}{0.9996} (x_{UTM} - 500{,}000) = -214{,}408.4314$$

$$y_{TM} = \frac{1}{0.9996} y_{UTM} = 4{,}760{,}058.8796$$

Step 2: Convert Transverse Mercator coordinates (x_{TM}, y_{TM}) to geodetic latitude and longitude(ϕ, λ).

2.1 Calculate the UTM zone central meridian (λ_0) using equation 7.23.

$$\lambda_0 = (6z + 177)\left(\frac{\pi}{180}\right) = 5.078908123303 \text{ radians}$$

2.2 Compute the eccentricity (ε) and second eccentricity (ε') of the Clarke 1866 ellipsoid, the reference ellipsoid for NAD 27, using equation set 7.9.

$$\varepsilon^2 = 2f - f^2 = 0.6768657997291 \times 10^{-2}$$

$$\varepsilon'^2 = \frac{\varepsilon^2}{1 - \varepsilon^2} = 0.6814784945915 \times 10^{-2}$$

2.3 Calculate the subscripted-A terms of equation set 7.9, using ε^2 calculated in Step 2.1. The subscripted-A terms are used in the calculation of the footpoint latitude.

$$A_0 = 1 - \frac{1}{4}\varepsilon^2 - \frac{3}{64}\varepsilon^4 - \frac{5}{256}\varepsilon^6 - \frac{175}{16384}\varepsilon^8 = 0.998305681856$$

$$A_2 = \frac{3}{8}\left(\varepsilon^2 + \frac{1}{4}\varepsilon^4 + \frac{15}{128}\varepsilon^6 - \frac{455}{4096}\varepsilon^8\right) = 0.2542555420215 \times 10^{-2}$$

$$A_4 = \frac{15}{256}\left(\varepsilon^4 + \frac{3}{4}\varepsilon^6 - \frac{77}{128}\varepsilon^8\right) = 0.2698010542695 \times 10^{-5}$$

$$A_6 = \frac{35}{3072}\left(\varepsilon^6 - \frac{41}{32}\varepsilon^8\right) = 0.3502448582027 \times 10^{-8}$$

$$A_8 = -\frac{315}{131072}\varepsilon^8 = -0.5044416191579 \times 10^{-11}$$

2.4 Compute the footpoint latitude, ϕ_1, using equation 7.10 and a from Table A.1.

Make an initial estimate for ϕ_1:

$$\phi_1 = \frac{y_{TM}}{a} = 0.7463005398433 \text{ radians}$$

2.5 Calculate S_ϕ using equation set 7.9 and the estimate of ϕ, ϕ_1.

$$S_\phi = a[A_0\phi - A_2 \sin 2\phi + A_4 \sin 4\phi - A_6 \sin 6\phi + A_8 \sin 8\phi]$$

$$S_\phi = 0.4735829138159 \text{ x } 10^7$$

2.6 Calculate S'_ϕ using equation set 7.11 and the estimate of ϕ, ϕ_1.

$$S'_\phi = a[A_0 - 2 A_2 \cos 2 \phi + 4 A_4 \cos 4\phi - 6 A_6 \cos 6\phi + 8 A_8 \cos 8\phi]$$

$$S'_\phi = 0.6364798134276 \text{ x } 10^7$$

2.7 Find an updated value of the footpoint latitude, ϕ_1 using equation 7.10.

$$\phi_1 \text{ (updated)} = \phi_1 \text{ (current)} - \frac{S_\phi - y}{S'_\phi} = 0.7501073756426 \text{ radians}$$

2.8 Iterate Steps 2.5 through 2.7 until successive values of ϕ_1 are as close as is desired.

For this example, the final value of ϕ_1 is:

Final $\phi_1 = 0.7501073020563$ radians

2.9 Calculate values used in equation set 7.12 using equation set 7.13.

$$t_1 = \tan \phi_1 = \tan (0.7501073020563)$$

$$N_1 = \frac{a}{\sqrt{1 - \varepsilon^2 \sin^2 \phi_1}} = 6,388,261.949762$$

$$R_1 = \frac{a (1 - \varepsilon^2)}{(1 - \varepsilon^2 \sin^2 \phi_1)^{3/2}} = 6,365,044.225886$$

$$\eta_1{}^2 = \varepsilon'^2 \cos^2 \phi_1 = 0.3647692467262 \times 10^{-2}$$

$$B_3 = 1 + 2\, t_1{}^2 + \eta_1{}^2 = 2.740138642336$$

$$B_4 = 5 + 3\, t_1{}^2 + \eta_1{}^2 - 4\, \eta_1{}^4 - 9\, t_1{}^2\, \eta_1{}^2 = 7.579827062321$$

$$B_5 = 5 + 28\, t_1{}^2 + 24\, t_1{}^4 + 6\, \eta_1{}^2 + 8\, t_1{}^2\, \eta_1{}^2 = 47.45050110665$$

$$B_6 = 61 + 90\, t_1{}^2 + 46\, \eta_1{}^2 + 45\, t_1{}^4 - 252\, t_1{}^2\, \eta_1{}^2 - 3\, \eta_1{}^4 - 66\, t_1{}^2\, \eta_1{}^4$$

$$- 90\, t_1{}^4\, \eta_1{}^2 + 225\, t_1{}^4\, \eta_1{}^4 = 172.1890097418$$

2.10   Calculate the geodetic latitude and longitude (ϕ, λ) using equation set 7.12.

$$\phi = \phi_1 - \frac{t_1\, N_1}{R_1} \left[\frac{1}{2} \left(\frac{x}{N_1} \right)^2 - \frac{B_4}{24} \left(\frac{x}{N_1} \right)^4 + \frac{B_6}{720} \left(\frac{x}{N_1} \right)^6 \right]$$

$$\phi = 0.74958094314009481 \text{ radians} = 42° \, 56' \, 52.168''$$

$$\lambda = \lambda_0 + \sec \phi_1 \left[\frac{x}{N_1} - \frac{B_3}{6} \left(\frac{x}{N_1} \right)^3 + \frac{B_5}{120} \left(\frac{x}{N_1} \right)^5 \right]$$

$$\lambda = 5.03305669671 \text{ radians} = 288° \, 22' \, 22.464''$$

8.4.6   Example 8.9. Convert WGS 84 latitude and longitude (ϕ, λ) to Lambert Conformal coordinates (x, y) using two standard parallels.

This example converts WGS 84 latitude and longitude to Lambert conformal projection coordinates in the WGS 84 reference system.

Beginning WGS 84 coordinates are

$$\phi_{\text{WGS 84}} = 42° \ 56' \, 52.163'' \, \text{N} = 0.749580918883376 \text{ radians}$$

$$\lambda_{\text{WGS 84}} = 288° \, 22' \, 24.326'' \, \text{E} = 5.033065722110382 \text{ radians}$$

Standard parallels:

$$\phi_1 \ = \ 42° \, 30' \, 00.000'' = 0.7417649320975901 \text{ radians}$$

$$\phi_2 \ = \ 43° \, 00' \, 00.000'' = 0.7504915783575617 \text{ radians}$$

Origin:

$$\phi_0 = 42° 30' 00.000'' = 0.7417649320975901 \text{ radians}$$

$$\lambda_0 = 288° 00' 00.000'' = 5.026548245743669 \text{ radians}$$

Step 1: Calculate eccentricity ε from flattening f.

$$\varepsilon = \sqrt{2f - f^2} = 0.0818191908426$$

Step 2: Calculate q at ϕ, ϕ_1, ϕ_2 and ϕ_0 using equation set 7.27.

$$q(\phi_i) = \ln\left[\tan\left(\frac{\pi}{4} + \frac{\phi_i}{2}\right) \cdot \left(\frac{1 - \varepsilon \sin \phi_i}{1 + \varepsilon \sin \phi_i}\right)^{\frac{\varepsilon}{2}} \right]$$

where

$$i \quad = \text{null}, 1, 2, 0$$

$$q(\phi) = 0.8270301921136922$$

$$q(\phi_1) = 0.8164293462094778$$

$$q(\phi_2) = 0.8282703592274435$$

$$q(\phi_0) = 0.8164293462094778$$

Step 3: Calculate N at ϕ_1 and ϕ_2 using equation set 7.27 and a from Table A.1.

$$N(\phi_i) = \frac{a}{\sqrt{1 - \varepsilon^2 \sin^2 \phi_i}}$$

$$N(\phi_1) = 6387903.46785173$$

$$N(\phi_2) = 6388090.05758619$$

Step 4:   Calculate L from equation set 7.27.

$$L = \frac{\ln(N_1 \cos \phi_1) - \ln(N_2 \cos \phi_2)}{q_2 - q_1} = 0.6788029306559900$$

Step 5: Calculate K from equation set 7.27.

$$K = \frac{N_1 \cos \phi_1}{L \, e^{-L \, q_1}} = \frac{N_2 \cos \phi_2}{L \, e^{-L \, q_2}} = 12076169.25203636$$

Step 6: Calculate r and r_0 from equation set 7.27.

$$r = K\, e^{-L\, q(\phi)} = 6888432.18075871$$

$$r_0 = K\, e^{-L\, q(\phi_0)} = 6938179.32109766$$

Step 7: Calculate the Lambert coordinates (x, y) using equation set 7.26.

$$x = r \sin L(\lambda - \lambda_0) = 30474.8898082$$

$$y = r_0 - r \cos L(\lambda - \lambda_0) = 49814.5521555$$

8.4.7 Example 8.10. Convert Lambert Conformal projection (with two standard parallels) coordinates (x, y) in the WGS 84 reference system to WGS 84 geodetic latitude and longitude (ϕ, λ).

This example converts WGS 84 Lambert Conformal coordinates to WGS 84 latitude and longitude.

Beginning Lambert Conformal coordinates, standard parallels (ϕ_1, ϕ_2) and origin (ϕ_0, λ_0) are

$$x = 30474.890$$

$$y = 49814.552$$

$$\phi_1 = 42° \ 30' \ 00'' = 0.7417649320976 \text{ radians}$$

$$\phi_2 = 43° \ 00' \ 00'' = 0.7504915783576 \text{ radians}$$

$$\phi_0 = 42° \ 30' \ 00'' = 0.7417649320976 \text{ radians}$$

$$\lambda_0 = 288° \ 00' \ 00'' = 5.0265482457437 \text{ radians}$$

Step 1: Compute ε, L, r_0, k, N_1, N_2, q_0, q_1, and q_2 using equation set 7.27. Obtain a from Table A.1.

$$\varepsilon = \sqrt{2f - f^2} = 0.0818191908426$$

$$q_i = \ln\left[\tan\left(\frac{\pi}{4} + \frac{\phi_i}{2}\right)\left(\frac{1 - \varepsilon \sin \phi_i}{1 + \varepsilon \sin \phi_i}\right)^{\frac{\varepsilon}{2}} \right]$$

$$q_0 = 0.8164293462094778$$

$$q_1 = 0.8164293462094778$$

$$q_2 = 0.8282703592274435$$

$$N_i = \frac{a}{\sqrt{1 - \varepsilon^2 \sin^2 \phi_i}}$$

$$N_1 = 6387903.467851727$$

$$N_2 = 6388090.057586191$$

$$L = \frac{\ln(N_1 \cos \phi_1) - \ln(N_2 \cos \phi_2)}{q_2 - q_1} = 0.6788029306559713$$

$$K = \frac{N_1 \cos \phi_1}{L\, e^{-Lq_1}} = \frac{N_2 \cos \phi_2}{L\, e^{-Lq_2}} = 12076169.25203651$$

Step 2:   Compute r_0 using equation set 7.27.

$$r_0 = K\, e^{-L\, q(\phi_0)} = 6938179.321097849$$

Step 3: Compute r and θ from equation set 7.29.

$$\theta = \arctan \frac{x}{r_0 - y} = 4.4240820859501107 \times 10^{-3} \text{ radians}$$

$$r = \frac{r_0 - y}{\cos\theta} = 6888432.18091526$$

Step 4: Compute geodetic longitude (λ) using equation set 7.28.

$$\lambda = \frac{\theta}{L} + \lambda_0 = 5.0330657221511255 \text{ radians} = 288° \, 22' \, 24.326''$$

Step 5: Compute q using equation set 7.28.

$$q = \frac{\ln \dfrac{K}{r}}{L} = 0.8270301920802524$$

Step 6: Find an initial approximation (ϕ') for the geodetic latitude from equation set 7.30.

$$\phi' = 2 \arctan(e^q) - \frac{\pi}{2} = 0.746233650926621$$

Step 7:   Compute $f(\phi')$ and $f'(\phi')$ from equation set 7.30.

$$f(\phi') = -q + \frac{1}{2}\ln\left[\left(\frac{1 + \sin\phi'}{1 - \sin\phi'}\right)\left(\frac{1 - \varepsilon\sin\phi'}{1 + \varepsilon\sin\phi'}\right)^{\varepsilon}\right] = -4.5493507336696604 \times 10^{-3}$$

$$f'(\phi') = \frac{1 - \varepsilon^2}{(1 - \varepsilon^2\sin^2\phi')\cos\phi'} = 1.357001593456580$$

Step 8:   Find an updated approximation (ϕ'') for the geodetic latitude using equation 7.31.

$$\phi'' = \phi' - \frac{f(\phi')}{f'(\phi')} = 0.7495861530575$$

Step 9: Iterate steps 7 and 8 until successive values of ϕ'' are sufficiently close after many iterations.

Final (ϕ) = 0.74958091885881052 radians = 42° 56′ 52.163″.

The resulting WGS 84 coordinates are:

$\phi_{\text{WGS 84}}$ = 42° 56′ 52.163″

$\lambda_{\text{WGS 84}}$ = 288° 22′ 24.326″

8.4.8 Example 8.11. Convert geodetic latitude and longitude (ϕ, λ) on the International
      Ellipsoid to Universal Polar Stereographic coordinates (x, y).

This example converts geodetic latitude and longitude to UPS projection coordinates. Coordinates are computed on the International Ellipsoid.

Beginning coordinates are

ϕ = 87° 17′ 14.400″ S = −1.523451361952464 radians

λ = 132° 14′ 52.303″ E = 2.308160619653466 radians

Step 1:   Compute the eccentricity (ε) of the International Ellipsoid using equation set 7.38 and
a and b from Table A.1.

$$\varepsilon^2 = 2f - f^2 = 0.0067226700623$$

$$\varepsilon = 0.08199189022$$

Step 2:   Compute K using equation set 7.38.

$$K = \frac{2a^2}{b}\left(\frac{1-\varepsilon}{1+\varepsilon}\right)^{\frac{\varepsilon}{2}} = 12713920.17558161$$

Step 3:   Compute r using equation set 7.38.

$$r = K \tan\left(\frac{\pi}{4} - \frac{|\phi|}{2}\right)\left(\frac{1+\varepsilon\sin|\phi|}{1-\varepsilon\sin|\phi|}\right)^{\frac{\varepsilon}{2}} = 303059.087879879$$

Step 4:   Compute UPS coordinates using equations 7.49 and 7.50.

$$x = 2{,}000{,}000 + 0.994\, r \sin\lambda = 2222991.410$$

$$y = 2{,}000{,}000 - 0.994\, r \cos\lambda = 1797464.051$$

8.4.9 Example 8.12. Convert Universal Polar Stereographic coordinates (x, y) to geodetic latitude and longitude (ϕ, λ).

This example converts UPS coordinates calculated on the International Ellipsoid to geodetic latitude and longitude on the International Ellipsoid.

Beginning UPS coordinates in the Southern hemisphere are

$$x = 2222991.410$$

$$y = 1797464.051$$

Step 1:   Convert Universal Polar Stereographic coordinates (X_{UPS}, Y_{UPS}) to Polar Stereographic coordinates (X_{PS}, Y_{PS}).

$$x_{PS} = \frac{x_{UPS} - 2000000}{0.994} = 224337.434205231$$

$$y_{PS} = \frac{y_{UPS} - 2000000}{0.994} = -203758.500100604$$

Step 2:   Compute the longitude (λ) from equation 7.39.

$$\lambda = \pi + \arctan\frac{x_{PS}}{y_{PS}}$$

$$\lambda = 2.308160619653466 \text{ radians} = 132°\ 14'\ 52.303''\ E$$

Step 3:   Compute k using equation 7.41, and a and b from Table A.1.

$$K = \frac{2a^2}{b}\left(\frac{1-\varepsilon}{1+\varepsilon}\right)^{\frac{\varepsilon}{2}} = 12713920.17558161$$

Step 4:   Compute q using equation set 7.40.

$$e^{-q} = \frac{|y|_{PS}}{K\,|\cos\lambda|}$$

q = 3.73652495299514

Step 5:   Compute an initial estimate (ϕ_0) of the geodetic latitude using equation set 7.42.

$$\phi_0 = 2\arctan e^q - \frac{\pi}{2} = 1.52313176628106 \text{ radians}$$

Step 6:   Compute f(ϕ_0) and f'(ϕ_0) using equation set 7.42.

$$f(\phi) = -q + \frac{1}{2}\,\ln\left[\left(\frac{1+\sin\phi}{1-\sin\phi}\right)\left(\frac{1-\varepsilon\sin\phi}{1+\varepsilon\sin\phi}\right)^{\varepsilon}\right]$$

f(ϕ_0) = −0.006730109061328

$$f'(\phi) = \frac{1-\varepsilon^2}{(1-\varepsilon^2\sin^2\phi)\cos\phi} = 20.9875717033244$$

Step 7:   Compute an updated geodetic latitude ϕ_1, using equation set 7.43.

$$\phi_n = \phi_{n-1} - \frac{f(\phi_{n-1})}{f'(\phi_{n-1})} = 1.523452437445710$$

Step 8:   Iterate steps 6 and 7 until successive values of the geodetic latitude are sufficiently close.

Final (ϕ) = 1.523451361952464 radians = 87° 17′ 14.4″S.

The resulting geodetic latitude and longitude (ϕ,λ) coordinates are:

ϕ = 87° 17′ 14.4″ S

λ = 132° 14′ 52.303″ E

8.5 List of test points. The test points listed here are taken from DMA TM 8358.2 (1989) and Snyder (1987).

Mercator
    Central Meridian = 180°
    Ellipsoid = Clarke 1866
    ϕ = 35° N
    λ = 75° W (285° E)

    x = 11,688,673.7 m
    y = 4,139,145.6 m
    k = 1.2194146

Universal Transverse Mercator
    Central Meridian = 285° E (75° W)
    Ellipsoid = Clarke 1866
    ϕ = 40° 30′ N
    λ = 73° 30′ W (286° 30′ E)

    x =   627,106.5 m
    y = 4,484,124.4 m
    k = 0.9997989

Lambert Conformal Conic
    Ellipsoid = Clarke 1866
    Standard parallels = 33° N
                                = 45° N
    Origin: ϕ_0 = 23° N
              λ_0 = 96° W (264° E)
    ϕ = 35° N
    λ = 75° W (285° E)

    x = 1,894,410.9 m
    y = 1,564,649.5 m
    k = 0.9970171

Universal Polar Stereographic
    Ellipsoid = International
    ϕ = 87° 17′ 14.400″ S
    λ = 132° 14′ 52.303″ E

    y = 1,797,464.051 m
    x = 2,222,991.410 m

APPENDIX A:
REFERENCE ELLIPSOID PARAMETERS

10. GENERAL

10.1 Scope. This appendix identifies reference ellipsoids with associated geometric parameters.

20. APPLICABLE DOCUMENTS.

The ellipsoids and associated parameters are taken from NGA TR8350.2 (2000).

While the information in this handbook is thought to be accurate, NGA TR8350.2 remains the authoritative source for ellipsoid parameters. Unless another edition is specified, the latest edition of NGA TR8350.2 should be used as the source for ellipsoid parameters when developing or updating software.

30. GENERAL REQUIREMENTS.

30.1   Application. The ellipsoid furnishes a simple, consistent, and uniform reference system
        for geodetic coordinates.

NOTE: Use semi-major axis a and flattening f to calculate ε and ε'. Do not use semi-minor axis b, which is a derived geometric constant.

TABLE A.1. Reference ellipsoids.

Reference Ellipsoid	ID Code	Semi-Major Axis a	Semi-Minor Axis b	Flattening f
Airy (1830)	AA	6377563.396	6356256.9090	1/299.3249646
Australian National	AN	6378160	6356774.7190	1/298.25
Bessel 1841				
Ethiopia, Indonesia, Japan, Korea	BR	6377397.155	6356078.9630	1/299.1528128
Namibia	BN	6377483.865	6356165.383	1/299.1528128
Clarke 1866	CC	6378206.4	6356583.800	1/294.9786982
Clarke 1880	CD	6378249.145	6356514.870	1/293.465
Everest				
Brunei & E. Malaysia (Sabah & Sarawak)	EB	6377298.556	6356097.550	1/300.8017
India 1830	EA	6377276.345	6356075.413	1/300.8017
India 1956***	EC	6377301.243	6356100.228	1/300.8017
Pakistan***	EF	6377309.613	6356109.571	1/300.8017
W. Malaysia and Singapore 1948	EE	6377304.063	6356103.039	1/300.8017
W. Malaysia 1969***	ED	6377295.664	6356094.668	1/300.8017
Geodetic Reference System 1980(GRS 80)	RF	6378137	6356752.3141	1/298.257222101
Helmert 1906	HE	6378200	6356818.170	1/298.3
Hough 1960	HO	6378270	6356794.343	1/297
Indonesian 1974	ID	6378160	6356774.504	1/298.247
International 1924	IN	6378388	6356911.946	1/297
Krassovsky 1940	KA	6378245	6356863.019	1/298.3
Modified Airy	AM	6377340.189	6356034.4480	1/299.3249646
Modified Fischer 1960 (South Asia)	FA	6378155	6356773.320	1/298.3
South American 1969	SA	6378160	6356774.719	1/298.25
World Geodetic System 1972 (WGS 72)	WD	6378135	6356750.520	1/298.26
World Geodetic System 1984 (WGS 84)	WE	6378137	6356752.3142	1/298.257223563

NOTE: All measurements are expressed in meters.

*** Through adoption of a new yard to meter conversion factor in the referenced country.   Ref: DMA TR 8350.2 (2nd edition, Insert 1).

APPENDIX B:
ΔX, ΔY AND ΔZ DATUM SHIFT PARAMETERS

10. GENERAL

10.1   Scope. Table B.1 of this appendix identifies the ellipsoid center shift transformation parameters used to transform a local geodetic system to WGS 84. An identification code, the reference ellipsoid and its identification code, the publication cycle, the first publication date of the parameter set, ΔX, ΔY and ΔZ, and the estimated errors (σ) in the shifts are listed for each local geodetic system.

20. APPLICABLE DOCUMENTS

The datum shift parameters and associated information for the most recent publication cycle of each local geodetic system are taken from NGA TR8350.2 (2000). The datum shift parameters for earlier publication cycles were taken from previous editions of DMA TR8350.2.

While the information in this handbook is thought to be accurate, NGA TR8350.2 remains the authoritative source for datum shift parameters. Unless another edition is specified, the latest edition of NGA TR8350.2 should be used as the source for datum shift parameters when developing or updating software.

30.   GENERAL REQUIREMENTS

30.1   Application. The ΔX, ΔY and ΔZ shifts are used for datum transformations using the three-step method or Molodensky methods discussed in section 6.3, and shown by equation sets 6.3, 6.4 and 6.5. Use of $\sigma_{\Delta X}$, $\sigma_{\Delta Y}$ and $\sigma_{\Delta Z}$ to estimate errors in geodetic coordinates attributable to shift parameters is discussed in Section 6.3.5.

There is more than one set of parameters for some reference ellipsoids. The applicable ellipsoid identification code (see Appendix A) is provided for each coverage area of each datum.

- -

The following symbology is used in Table B.1.

*   WGS 84 minus local geodetic system.

**   Derived from non-satellite sources, accuracy not specified.

TABLE B.1. Ellipsoid center shift transformation parameters, local geodetic system to WGS 84.

Local Geodetic System		Reference Ellipsoid	Publication		Ellipsoid Center Shift (meters) *					
Name	Code	(ID Code)	Cycle	Date	ΔX	$\sigma_{\Delta X}$	ΔY	$\sigma_{\Delta Y}$	ΔZ	$\sigma_{\Delta Z}$
ADINDAN	ADI	Clarke 1880								
Mean Solution (Ethiopia and Sudan)	ADI-M	(CD)	0	1991	-166	5	-15	5	204	3
Burkina Faso	ADI-E	(CD)	0	1991	-118	25	-14	25	218	25
Cameroon	ADI-F	(CD)	0	1991	-134	25	-2	25	210	25
Ethiopia	ADI-A	(CD)	0	1991	-165	3	-11	3	206	3
Mali	ADI-C	(CD)	0	1991	-123	25	-20	25	220	25
Senegal	ADI-D	(CD)	0	1991	-128	25	-18	25	224	25
Sudan	ADI-B	(CD)	0	1991	-161	3	-14	5	205	3
AFGOOYE	AFG	Krassovsky 1940								
Somalia		(KA)	0	1987	-43	25	-163	25	45	25
AIN EL ABD 1970	AIN	International 1924								
Bahrain Island	AIN-A	(IN)	0	1991	-150	25	-250	25	-1	25
Saudi Arabia	AIN-B	(IN)	0	1991	-143	10	-236	10	7	10
AMERICAN SAMOA 1962	AMA	Clarke 1866								
American Somoa Islands		(CC)	0	1993	-115	25	118	25	426	25
ANNA 1 ASTRO 1965	ANO	Australian National								
Cocos Islands		(AN)	0	1987	-491	25	-22	25	435	25
ANTIGUA ISLAND ASTRO 1943	AIA	Clarke 1880								
Antigua, Leeward Islands		(CD)	0	1991	-270	25	13	25	62	25
ARC 1950	ARF	Clarke 1880								
Mean Solution (Botswana, Lesotho, Malawi, Swaziland, Zaire, Zambia, and Zimbabwe)	ARF-M	(CD)	0	1987	-143	20	-90	33	-294	20
Botswana	ARF-A	(CD)	0	1991	-138	3	-105	5	-289	3
Burundi	ARF-H	(CD)	0	1991	-153	20	-5	20	-292	20

TABLE B.1. Ellipsoid center shift transformation parameters, local geodetic system to WGS 84.

Local Geodetic System		Reference Ellipsoid	Publication		Ellipsoid Center Shift (meters) *					
Name	Code	(ID Code)	Cycle	Date	ΔX	$\sigma_{\Delta X}$	ΔY	$\sigma_{\Delta Y}$	ΔZ	$\sigma_{\Delta Z}$
Lesotho	ARF-B	(CD)	0	1991	-125	3	-108	3	-295	8
Malawi	ARF-C	(CD)	0	1991	-161	9	-73	24	-317	8
Swaziland	ARF-D	(CD)	0	1991	-134	15	-105	15	-295	15
Zaire	ARF-E	(CD)	0	1991	-169	25	-19	25	-278	25
Zambia	ARF-F	(CD)	0	1991	-147	21	-74	21	-283	27
Zimbabwe	ARF-G	(CD)	0	1991	-142	5	-96	8	-293	11
ARC 1960	ARS	Clarke 1880								
Mean Solution (Kenya and Tanzania)	ARS-M	(CD)	0	1991	-160	20	-6	20	-302	20
Kenya	ARS-A	(CD)	0	1997	-157	4	-2	3	-299	3
Tanzania	ARS-B	(CD)	0	1997	-175	6	-23	9	-303	10
ASCENSION ISLAND 1958	ASC	International 1924								
Ascension Island		(IN)	0	1991	-205	25	107	25	53	25
ASTRO BEACON "E" 1945	ATF	International 1924								
Iwo Jima		(IN)	0	1987	145	25	75	25	-272	25
ASTRO DOS 71/4	SHB	International 1924								
St. Helena Island		(IN)	0	1987	-320	25	550	25	-494	25
ASTRONOMICAL STATION 1952	ASQ	International 1924								
Marcus Island		(IN)	0	1987	124	25	-234	25	-25	25
ASTRO TERN ISLAND (FRIG) 1961	TRN	International 1924								
Tern Island		(IN)	0	1991	114	25	-116	25	-333	25
AUSTRALIAN GEODETIC 1966	AUA	Australian National								
Australia and Tasmania		(AN)	0	1987	-133	3	-48	3	148	3

TABLE B.1. Ellipsoid center shift transformation parameters, local geodetic system to WGS 84.

Local Geodetic System		Reference Ellipsoid (ID Code)	Publication		Ellipsoid Center Shift (meters) *					
Name	Code		Cycle	Date	ΔX	$\sigma_{\Delta X}$	ΔY	$\sigma_{\Delta Y}$	ΔZ	$\sigma_{\Delta Z}$
AUSTRALIAN GEODETIC 1984	AUG	Australian National								
Australia and Tasmania		(AN)	0	1987	-134	2	-48	2	149	2
AYABELLE LIGHTHOUSE	PHA	Clarke 1880								
Djibouti		(CD)	0	1991	-79	25	-129	25	145	25
BELLEVUE (IGN)	IBE	International 1924								
Efate and Erromango Islands		(IN)	0	1987	-127	20	-769	20	472	20
BERMUDA 1957	BER	Clarke 1866								
Bermuda Islands		(CC)	0	1987	-73	20	213	20	296	20
BISSAU	BID	International 1924								
Guinea-Bissau		(IN)	0	1991	-173	25	253	25	27	25
BOGOTA OBSERVATORY	BOO	International 1924								
Colombia		(IN)	0	1987	307	6	304	5	-318	6
BUKIT RIMPAH	BUR	Bessel 1841								
Bangka and Belitung Islands (Indonesia)		(BR)	0	1987	-384	**	664	**	-48	**
CAMP AREA ASTRO	CAZ	International 1924								
Camp McMurdo Area, Antarctica		(IN)	0	1987	-104	**	-129	**	239	**
CAMPO INCHAUSPE 1969	CAI	International 1924								
Argentina		(IN)	0	1987	-148	5	136	5	90	5
CANTON ASTRO 1966	CAO	International 1924								
Phoenix Islands		(IN)	0	1987	298	15	-304	15	-375	15
CAPE	CAP	Clarke 1880								

TABLE B.1. Ellipsoid center shift transformation parameters, local geodetic system to WGS 84.

Local Geodetic System		Reference Ellipsoid (ID Code)	Publication		Ellipsoid Center Shift (meters) *					
Name	Code		Cycle	Date	ΔX	$\sigma_{\Delta X}$	ΔY	$\sigma_{\Delta Y}$	ΔZ	$\sigma_{\Delta Z}$
South Africa		(CD)	0	1987	-136	3	-108	6	-292	6
CAPE CANAVERAL Mean Solution (Florida and Bahamas)	CAC	Clarke 1866 (CC)	0	1991	-2	3	151	3	181	3
CARTHAGE Tunisia	CGE	Clarke 1880 (CD)	0	1987	-263	6	6	9	431	8
CHATHAM ISLAND ASTRO 1971 Chatham Island (New Zealand)	CHI	International 1924 (IN)	0	1987	175	15	-38	15	113	15
CHUA ASTRO Paraguay	CHU	International 1924 (IN)	0	1987	-134	6	229	9	-29	5
CO-ORDINATE SYSTEM 1937 OF ESTONIA Estonia	EST	Bessel 1841 (BR)	0	1997	374	2	150	3	588	3
CORREGO ALEGRE Brazil	COA	International 1924 (IN)	0	1987	-206	5	172	3	-6	5
DABOLA Guinea	DAL	Clarke 1880 (CD)	0	1991	-83	15	37	15	124	15
DJAKARTA (BATAVIA) Sumatra (Indonesia)	BAT	Bessel 1841 (BR)	0	1987	-377	3	681	3	-50	3
DECEPTION ISLAND Deception Island, Antarctica	DID	Clarke 1880 (CD)	0	1993	260	20	12	20	-147	20
DOS 1968 Gizo Island (New Georgia Islands)	GIZ	International 1924 (IN)	0	1987	230	25	-199	25	-752	25
EASTER ISLAND 1967	EAS	International 1924								

TABLE B.1. Ellipsoid center shift transformation parameters, local geodetic system to WGS 84.

Local Geodetic System		Reference Ellipsoid	Publication		Ellipsoid Center Shift (meters) *					
Name	Code	(ID Code)	Cycle	Date	ΔX	$\sigma_{\Delta X}$	ΔY	$\sigma_{\Delta Y}$	ΔZ	$\sigma_{\Delta Z}$
Easter Island		(IN)	0	1987	211	25	147	25	111	25
EUROPEAN 1950	EUR	International 1924								
Mean Solution (Austria, Belgium, Denmark, Finland, France, FRG (Federal Republic of Germany) (before 1 January 1993), Gibraltar, Greece, Italy, Luxembourg, Netherlands, Norway, Portugal, Spain, Sweden, and Switzerland)	EUR-M	(IN)	0	1987	-87	3	-98	8	-121	5
Western Europe (Limited to Austria, Denmark, France, Federal Republic of Germany (before 1 January 1993), Netherlands, and Switzerland)	EUR-A	(IN)	0	1991	-87	3	-96	3	-120	3
Cyprus	EUR-E	(IN)	0	1991	-104	15	-101	15	-140	15
Egypt	EUR-F	(IN)	0	1991	-130	6	-117	8	-151	8
England, Channel Islands, Scotland, and Shetland Islands	EUR-G	(IN)	0	1991	-86	3	-96	3	-120	3
England, Ireland, Scotland, and Shetland Islands	EUR-K	(IN)	0	1991	-86	3	-96	3	-120	3
Greece	EUR-B	(IN)	0	1991	-84	25	-95	25	-130	25
Iran	EUR-H	(IN)	0	1991	-117	9	-132	12	-164	11
Iraq, Israel, Jordan, Kuwait, Lebanon, Saudi Arabia, and Syria	EUR-S	(IN)	0	1991	-103	**	-106	**	-141	**
Sardinia (Italy)	EUR-I	(IN)	0	1991	-97	25	-103	25	-120	25
Sicily (Italy)	EUR-J	(IN)	0	1991	-97	20	-88	20	-135	20
Malta	EUR-L	(IN)	0	1991	-107	25	-88	25	-149	25
Norway and Finland	EUR-C	(IN)	0	1991	-87	3	-95	5	-120	3
Portugal and Spain	EUR-D	(IN)	0	1991	-84	5	-107	6	-120	3
Tunisia	EUR-T	(IN)	0	1993	-112	25	-77	25	-145	25
EUROPEAN 1979	EUS	International 1924								

TABLE B.1. Ellipsoid center shift transformation parameters, local geodetic system to WGS 84.

Local Geodetic System		Reference Ellipsoid (ID Code)	Publication		Ellipsoid Center Shift (meters) *					
Name	Code		Cycle	Date	ΔX	$\sigma_{\Delta X}$	ΔY	$\sigma_{\Delta Y}$	ΔZ	$\sigma_{\Delta Z}$
Mean Solution: Austria, Finland, Netherlands, Norway, Spain, Sweden, and Switzerland		(IN)	0	1987	-86	3	-98	3	-119	3
FORT THOMAS 1955 Nevis, St. Kitts, Leeward Islands	FOT	Clarke 1880 (CD)	0	1991	-7	25	215	25	225	25
GAN 1970 Republic of Maldives	GAA	International 1924 (IN)	0	1987	-133	25	-321	25	50	25
GEODETIC DATUM 1949 New Zealand	GEO	International 1924 (IN)	0	1987	84	5	-22	3	209	5
GRACIOSA BASE SW 1948 Faial, Graciosa, Pico, Sao Jorge, and Terceira Islands (Azores)	GRA	International 1924 (IN)	0	1991	-104	3	167	3	-38	3
GUAM 1963 Guam Island	GUA	Clarke 1866 (CC)	0	1987	-100	3	-248	3	259	3
GUNUNG SEGARA Kalimantan Island (Indonesia)	GSE	Bessel 1841 (BR)	0	1987	-403	**	684	**	41	**
GUX 1 ASTRO Guadalcanal Island	DOB	International 1924 (IN)	0	1987	252	25	-209	25	-751	25
HERAT NORTH Afghanistan	HEN	International 1924 (IN)	0	1987	-333	**	-222	**	114	**
HERMANNSKOGEL Yugoslavia (before 1990), Slovenia, Croatia, Bosnia and Herzegovina, Serbia	HER	Bessel 1841 (BR)	0	1997	682	**	-203	**	480	**

TABLE B.1. Ellipsoid center shift transformation parameters, local geodetic system to WGS 84.

Local Geodetic System		Reference Ellipsoid (ID Code)	Publication		Ellipsoid Center Shift (meters) *					
Name	Code		Cycle	Date	ΔX	$\sigma_{\Delta X}$	ΔY	$\sigma_{\Delta Y}$	ΔZ	$\sigma_{\Delta Z}$
HJORSEY 1955	HJO	International 1924								
Iceland		(IN)	0	1987	-73	3	46	3	-86	6
HONG KONG 1963	HKD	International 1924								
Hong Kong		(IN)	0	1987	-156	25	-271	25	-189	25
HU-TZU-SHAN	HTN	International 1924								
Taiwan		(IN)	0	1991	-637	15	-549	15	-203	15
INDIAN	IND	Everest								
Bangladesh	IND-B	(EA)	0	1991	282	10	726	8	254	12
India and Nepal	IND-I	(EC)	0	1991	295	12	736	10	257	15
Pakistan	IND-P	(EF)	0	1993	283	**	682	**	231	**
INDIAN 1954	INF	Everest								
Thailand	INF-A	(EA)	0	1993	217	15	823	6	299	12
INDIAN 1960	ING	Everest								
Vietnam (near 16 deg. N)	ING-A	(EA)	0	1993	198	25	881	25	317	25
Con Son Island (Vietnam)	ING-B	(EA)	0	1993	182	25	915	25	344	25
INDIAN 1975	INH	Everest								
Thailand	INH-A	(EA)	0	1991	209	12	818	10	290	12
Thailand	INH-A1	(EA)	1	1997	210	3	814	2	289	3
INDONESIAN 1974	IDN	Indonesian 1974								
Indonesia		(ID)	0	1993	-24	25	-15	25	5	25
IRELAND 1965	IRL	Modified Airy								
Ireland		(AM)	0	1987	506	3	-122	3	611	3
ISTS 061 ASTRO 1968	ISG	International 1924								

TABLE B.1. Ellipsoid center shift transformation parameters, local geodetic system to WGS 84.

Local Geodetic System		Reference Ellipsoid (ID Code)	Publication		Ellipsoid Center Shift (meters) *					
Name	Code		Cycle	Date	ΔX	$\sigma_{\Delta X}$	ΔY	$\sigma_{\Delta Y}$	ΔZ	$\sigma_{\Delta Z}$
South Georgia Island		(IN)	0	1991	-794	25	119	25	-298	25
ISTS 073 ASTRO 1969	IST	International 1924								
Diego Garcia		(IN)	0	1987	208	25	-435	25	-229	25
JOHNSTON ISLAND 1961	JOH	International 1924								
Johnston Island		(IN)	0	1991	189	25	-79	25	-202	25
KANDAWALA	KAN	Everest								
Sri Lanka		(EA)	0	1987	-97	20	787	20	86	20
KERGUELEN ISLAND 1949	KEG	International 1924								
Kerguelen Island		(IN)	0	1987	145	25	-187	25	103	25
KERTAU 1948	KEA	Everest								
West Malaysia and Singapore		(EE)	0	1987	-11	10	851	8	5	6
KOREAN GEODETIC DATUM 1995	KGS	WGS 84								
South Korea		(WE)	0	1997	0	1	0	1	0	1
KUSAIE ASTRO 1951	KUS	International 1924								
Caroline Islands, Fed. States of Micronesia		(IN)	0	1991	647	25	1777	25	-1124	25
L.C. 5 ASTRO 1961	LCF	Clarke 1866								
Cayman Brac Island		(CC)	0	1987	42	25	124	25	147	25
LEIGON	LEH	Clarke 1880								
Ghana		(CD)	0	1991	-130	2	29	3	364	2
LIBERIA 1964	LIB	Clarke 1880								
Liberia		(CD)	0	1987	-90	15	40	15	88	15
LUZON	LUZ	Clarke 1866								
Philippines (Excluding Mindanao Island)	LUZ-A	(CC)	0	1987	-133	8	-77	11	-51	9
Mindanao Island	LUZ-B	(CC)	0	1987	-133	25	-79	25	-72	25

TABLE B.1. Ellipsoid center shift transformation parameters, local geodetic system to WGS 84.

Local Geodetic System		Reference Ellipsoid (ID Code)	Publication		Ellipsoid Center Shift (meters) *					
Name	Code		Cycle	Date	ΔX	$\sigma_{\Delta X}$	ΔY	$\sigma_{\Delta Y}$	ΔZ	$\sigma_{\Delta Z}$
MAHE 1971	MIK	Clarke 1880								
Mahe Island		(CD)	0	1987	41	25	-220	25	-134	25
MASSAWA	MAS	Bessel 1841								
Eritrea (Ethiopia)		(BR)	0	1987	639	25	405	25	60	25
MERCHICH	MER	Clarke 1880								
Morocco		(CD)	0	1987	31	5	146	3	47	3
MIDWAY ASTRO 1961	MID	International 1924								
Midway Islands		(IN)	0	1987	912	25	-58	25	1227	25
MINNA	MIN	Clarke 1880								
Cameroon	MIN-A	(CD)	0	1991	-81	25	-84	25	115	25
Nigeria	MIN-B	(CD)	0	1987	-92	3	-93	6	122	5
MONTSERRAT ISLAND ASTRO 1958	ASM	Clarke 1880								
Montserrat, Leeward Islands		(CD)	0	1991	174	25	359	25	365	25
M'PORALOKO	MPO	Clarke 1880								
Gabon		(CD)	0	1991	-74	25	-130	25	42	25
NAHRWAN	NAH	Clarke 1880								
Masirah Island (Oman)	NAH-A	(CD)	0	1987	-247	25	-148	25	369	25
United Arab Emirates	NAH-B	(CD)	0	1987	-249	25	-156	25	381	25
Saudi Arabia	NAH-C	(CD)	0	1991	-243	20	-192	20	477	20
NAPARIMA, BWI	NAP	International 1924								
Trinidad and Tobago		(IN)	0	1991	-10	15	375	15	165	15
NORTH AMERICAN 1927	NAS	Clarke 1866								
Mean Solution (CONUS)	NAS-C	(CC)	0	1987	-8	5	160	5	176	6

TABLE B.1. Ellipsoid center shift transformation parameters, local geodetic system to WGS 84.

Local Geodetic System		Reference Ellipsoid	Publication		Ellipsoid Center Shift (meters) *					
Name	Code	(ID Code)	Cycle	Date	ΔX	$\sigma_{\Delta X}$	ΔY	$\sigma_{\Delta Y}$	ΔZ	$\sigma_{\Delta Z}$
Western United States (Arizona, Arkansas, California, Colorado, Idaho, Iowa, Kansas, Montana, Nebraska, Nevada, New Mexico, North Dakota, Oklahoma, Oregon, South Dakota, Texas, Utah, Washington, and Wyoming)	NAS-B	(CC)	0	1991	-8	5	159	3	175	3
Eastern United States (Alabama, Connecticut, Delaware, District of Columbia, Florida, Georgia, Illinois, Indiana, Kentucky, Louisiana, Maine, Maryland, Massachusetts, Michigan, Minnesota, Mississippi, Missouri, New Hampshire, New Jersey, New York, North Carolina, Ohio, Pennsylvania, Rhode Island, South Carolina, Tennessee, Vermont, Virginia, West Virginia, and Wisconsin)	NAS-A	(CC)	0	1991	-9	5	161	5	179	8
Alaska (Excluding Aleutian Islands)	NAS-D	(CC)	0	1987	-5	5	135	9	172	5
Aleutian Islands East of 180°W	NAS-V	(CC)	0	1993	-2	6	152	8	149	10
Aleutian Islands West of 180°W	NAS-W	(CC)	0	1993	2	10	204	10	105	10
Bahamas (Excluding San Salvador Island)	NAS-Q	(CC)	0	1987	-4	5	154	3	178	5
San Salvador Island	NAS-R	(CC)	0	1987	1	25	140	25	165	25
Canada (Mean Solution, including Newfoundland)	NAS-E	(CC)	0	1987	-10	15	158	11	187	6
Alberta and British Columbia	NAS-F	(CC)	0	1991	-7	8	162	8	188	6
Eastern Canada (Newfoundland, New Brunswick, Nova Scotia and Quebec)	NAS-G	(CC)	0	1991	-22	6	160	6	190	3
Manitoba and Ontario	NAS-H	(CC)	0	1991	-9	9	157	5	184	5
Northwest Territories and Saskatchwan	NAS-I	(CC)	0	1991	4	5	159	5	188	3
Yukon	NAS-J	(CC)	0	1991	-7	5	139	8	181	3
Canal Zone	NAS-O	(CC)	0	1987	0	20	125	20	201	20
Caribbean (Antigua Island, Barbados, Barbuda, Caicos Islands, Cuba, Dominican Republic, Grand Cayman, Jamaica, and Turks Islands)	NAS-P	(CC)	0	1991	-3	3	142	9	183	12

TABLE B.1. Ellipsoid center shift transformation parameters, local geodetic system to WGS 84.

Local Geodetic System		Reference Ellipsoid (ID Code)	Publication		Ellipsoid Center Shift (meters) *					
Name	Code		Cycle	Date	ΔX	$\sigma_{\Delta X}$	ΔY	$\sigma_{\Delta Y}$	ΔZ	$\sigma_{\Delta Z}$
Central America (Belize, Costa Rica, El Salvador, Guatemala, Honduras, and Nicaragua)	NAS- N	(CC)	0	1987	0	8	125	3	194	5
Cuba	NAS-T	(CC)	0	1987	-9	25	152	25	178	25
Greenland (Hayes Peninsula)	NAS-U	(CC)	0	1987	11	25	114	25	195	25
Mexico	NAS-L	(CC)	0	1987	-12	8	130	6	190	6
NORTH AMERICAN 1983	NAR	GRS 80								
Alaska (Excluding Aleutian Islands)	NAR-A	(RF)	0	1987	0	2	0	2	0	2
Aleutian Islands	NAR-E	(RF)	0	1993	-2	5	0	2	4	5
Canada	NAR-B	(RF)	0	1987	0	2	0	2	0	2
CONUS	NAR-C	(RF)	0	1987	0	2	0	2	0	2
Hawaii	NAR-H	(RF)	0	1993	1	2	1	2	-1	2
Mexico and Central America	NAR-D	(RF)	0	1987	0	2	0	2	0	2
NORTH SAHARA 1959	NSD	Clarke 1880								
Algeria		(CD)	0	1993	-186	25	-93	25	310	25
OBSERVATORIO METEROLOGICO 1939	FLO	International 1924								
Corvo and Flores Islands (Azores)		(IN)	0	1991	-425	20	-169	20	81	20
OLD EGYPTIAN 1907	OEG	Helmert 1906								
Egypt		(HE)	0	1987	-130	3	110	6	-13	8
OLD HAWAIIAN (**See note below**)	OHA	Clarke 1866								
Mean Solution	OHA-M	(CC)	0	1987	61	25	-285	20	-181	20
Hawaii	OHA-A	(CC)	0	1991	89	25	-279	25	-183	25
Kauai	OHA-B	(CC)	0	1991	45	20	-290	20	-172	20
Maui	OHA-C	(CC)	0	1991	65	25	-290	25	-190	25
Oahu	OHA-D	(CC)	0	1991	58	10	-283	6	-182	6

TABLE B.1. Ellipsoid center shift transformation parameters, local geodetic system to WGS 84.

Local Geodetic System		Reference Ellipsoid	Publication		Ellipsoid Center Shift (meters) *					
Name	Code	(ID Code)	Cycle	Date	ΔX	$\sigma_{\Delta X}$	ΔY	$\sigma_{\Delta Y}$	ΔZ	$\sigma_{\Delta Z}$
(**NOTE:** Old Hawaiian datum coordinates may be found on both the International 1924 and Clarke 1866 ellipsoids. See Appendix F for transformation of Old Hawaiian datum coordinates on the International 1924 ellipsoid.)										
OMAN	FAH	Clarke 1880								
Oman		(CD)	0	1987	-346	3	-1	3	224	9
ORDNANCE SURVEY OF GREAT BRITAIN 1936	OGB	Airy								
Mean Solution (England, Isle of Man, Scotland, Shetland Islands, and Wales)	OGB-M	(AA)	0	1987	375	10	-111	10	431	15
England	OGB-A	(AA)	0	1991	371	5	-112	5	434	6
England, Isle of Man, and Wales	OGB-B	(AA)	0	1991	371	10	-111	10	434	15
Scotland and Shetland Island	OGB-C	(AA)	0	1991	384	10	-111	10	425	10
Wales	OGB-D	(AA)	0	1991	370	20	-108	20	434	20
PICO DE LAS NIEVES	PLN	International 1924								
Canary Islands		(IN)	0	1987	-307	25	-92	25	127	25
PITCAIRN ASTRO 1967	PIT	International 1924								
Pitcairn Island		(IN)	0	1987	185	25	165	25	42	25
POINT 58	PTB	Clarke 1880								
Mean Solution (Burkina Faso and Niger)		(CD)	0	1991	-106	25	-129	25	165	25
POINTE NOIRE 1948	PTN	Clarke 1880								
Congo		(CD)	0	1991	-148	25	51	25	-291	25
PORTO SANTO 1936	POS	International 1924								
Porto Santo and Madeira Islands		(IN)	0	1991	-499	25	-249	25	314	25
PROVISIONAL SOUTH AMERICAN 1956	PRP	International 1924								

TABLE B.1. Ellipsoid center shift transformation parameters, local geodetic system to WGS 84.

Local Geodetic System		Reference Ellipsoid	Publication		Ellipsoid Center Shift (meters) *					
Name	Code	(ID Code)	Cycle	Date	ΔX	$\sigma_{\Delta X}$	ΔY	$\sigma_{\Delta Y}$	ΔZ	$\sigma_{\Delta Z}$
Mean Solution (Bolivia, Chile, Colombia, Ecuador, Guyana, Peru, and Venezuela)	PRP-M	(IN)	0	1987	-288	17	175	27	-376	27
Bolivia	PRP-A	(IN)	0	1991	-270	5	188	11	-388	14
Chile (Northern Chile near 19ºS)	PRP-B	(IN)	0	1991	-270	25	183	25	-390	25
Chile (Southern Chile near 43ºS)	PRP-C	(IN)	0	1991	-305	20	243	20	-442	20
Colombia	PRP-D	(IN)	0	1991	-282	15	169	15	-371	15
Ecuador	PRP-E	(IN)	0	1991	-278	3	171	5	-367	3
Guyana	PRP-F	(IN)	0	1991	-298	6	159	14	-369	5
Peru	PRP-G	(IN)	0	1991	-279	6	175	8	-379	12
Venezuela	PRP-H	(IN)	0	1991	-295	9	173	14	-371	15
PROVISIONAL SOUTH CHILEAN 1963 (also known as Hito XVIII 1963)	HIT	International 1924								
Southern Chile near 53ºS		(IN)	0	1987	16	25	196º	25	93	25
PUERTO RICO	PUR	Clarke 1866								
Puerto Rico and Virgin Islands		(CC)	0	1987	11	3	72	3	-101	3
PULKOVO 1942	PUK	Krassovsky 1940								
Russia		(KA)	0	1993	28	**	-130	**	-95	**
QATAR NATIONAL	QAT	International 1924								
Qatar		(IN)	0	1987	-128	20	-283	20	22	20
QORNOQ	QUO	International 1924								
South Greenland		(IN)	0	1987	164	25	138	25	-189	32
REUNION	REU	International 1924								
Mascarene Islands		(IN)	0	1987	94	25	-948	25	-1262	25

TABLE B.1. Ellipsoid center shift transformation parameters, local geodetic system to WGS 84.

Local Geodetic System		Reference Ellipsoid	Publication		Ellipsoid Center Shift (meters) *					
Name	Code	(ID Code)	Cycle	Date	ΔX	$\sigma_{\Delta X}$	ΔY	$\sigma_{\Delta Y}$	ΔZ	$\sigma_{\Delta Z}$
ROME 1940	MOD	International 1924								
Sardinia		(IN)	0	1987	-225	25	-65	25	9	25
S-42 (PULKOVO 1942)	SPK	Krassovsky 1940								
Hungary	SPK-A	(KA)	0	1993	28	2	-121	2	-77	2
Poland	SPK-B	(KA)	0	1997	23	4	-124	2	-82	4
Czechoslovakia (Before 1 January 1993)	SPK-C	(KA)	0	1997	26	3	-121	3	-78	2
Latvia	SPK-D	(KA)	0	1997	24	2	-124	2	-82	2
Kazakhstan	SPK-E	(KA)	0	1997	15	25	-130	25	-84	25
Albania	SPK-F	(KA)	0	1997	24	3	-130	3	-92	3
Romania	SPK-G	(KA)	0	1997	28	3	-121	5	-77	3
S-JTSK	CCD	Bessel 1841								
Czechoslovakia (Before 1 January 1993)		(BR)	0	1993	589	4	76	2	480	3
SANTO (DOS) 1965	SAE	International 1924								
Espirito Santo Island		(IN)	0	1987	170	25	42	25	84	25
SAO BRAZ	SAO	International 1924								
Sao Miguel, Santa Maria Islands (Azores)		(IN)	0	1987	-203	25	141	25	53	25
SAPPER HILL 1943	SAP	International 1924								
East Falkland Island		(IN)	0	1991	-355	1	21	1	72	1
SCHWARZECK	SCK	Bessel 1841								
Namibia		(BN)	0	1991	616	20	97	20	-251	20
SELVAGEM GRANDE 1938	SGM	International 1924								
Salvage Islands		(IN)	0	1991	-289	25	-124	25	60	25

TABLE B.1. Ellipsoid center shift transformation parameters, local geodetic system to WGS 84.

Local Geodetic System		Reference Ellipsoid	Publication		Ellipsoid Center Shift (meters) *					
Name	Code	(ID Code)	Cycle	Date	ΔX	$\sigma_{\Delta X}$	ΔY	$\sigma_{\Delta Y}$	ΔZ	$\sigma_{\Delta Z}$
SIERRA LEONE 1960	SRL	Clark 1880								
Sierra Leone		(CD)	0	1997	-88	15	4	15	101	15
SOUTH AMERICAN 1969	SAN	S. American 1969								
Mean Solution (Argentina, Bolivia, Brazil, Chile, Colombia, Ecuador, Guyana, Paraguay, Peru, Trinidad and Tobago, and Venezuela)	SAN-M	(SA)	0	1987	-57	15	1	6	-41	9
Argentina	SAN-A	(SA)	0	1991	-62	5	-1	5	-37	5
Baltra, Galapagos Islands	SAN-J	(SA)	0	1991	-47	25	26	25	-42	25
Bolivia	SAN-B	(SA)	0	1991	-61	15	2	15	-48	15
Brazil	SAN-C	(SA)	0	1991	-60	3	-2	5	-41	5
Chile	SAN-D	(SA)	0	1991	-75	15	-1	8	-44	11
Colombia	SAN-E	(SA)	0	1991	-44	6	6	6	-36	5
Ecuador (excluding Galapagos Islands)	SAN-F	(SA)	0	1991	-48	3	3	3	-44	3
Guyana	SAN-G	(SA)	0	1991	-53	9	3	5	-47	5
Paraguay	SAN-H	(SA)	0	1991	-61	15	2	15	-33	15
Peru	SAN-I	(SA)	0	1991	-58	5	0	5	-44	5
Trinidad and Tobago	SAN-K	(SA)	0	1991	-45	25	12	25	-33	25
Venezuela	SAN-L	(SA)	0	1991	-45	3	8	6	-33	3
SOUTH AMERICAN GEOCENTRIC REFERENCE SYSTEM (SIRGAS)	SIR	GRS 80								
South America		(RF)	0	1997	0	1	0	1	0	1
SOUTH ASIA	SOA	Modified Fischer 1960								
Singapore		(FA)	0	1987	7	25	-10	25	-26	25
TANANARIVE OBSERVATORY 1925	TAN	International 1924								
Madagascar		(IN)	0	1987	-189	**	-242	**	-91	**

TABLE B.1. Ellipsoid center shift transformation parameters, local geodetic system to WGS 84.

Local Geodetic System		Reference Ellipsoid (ID Code)	Publication		Ellipsoid Center Shift (meters) *					
Name	Code		Cycle	Date	ΔX	$\sigma_{\Delta X}$	ΔY	$\sigma_{\Delta Y}$	ΔZ	$\sigma_{\Delta Z}$
TIMBALAI 1948	TIL	Everest								
Brunei and East Malaysia (Sarawak and Sabah)		(EB)	0	1987	-679	10	669	10	-48	12
TOKYO	TOY	Bessel 1841								
Mean Solution (Japan, Okinawa, and South Korea)	TOY-M	(BR)	0	1991	-148	20	507	5	685	20
Japan	TOY-A	(BR)	0	1991	-148	8	507	5	685	8
Okinawa	TOY-C	(BR)	0	1991	-158	20	507	5	676	20
South Korea	TOY-B	(BR)	0	1991	-146	8	507	5	687	8
South Korea	TOY-B1	(BR)	1	1997	-147	2	506	2	687	2
TRISTAN ASTRO 1968	TDC	International 1924								
Tristan da Cunha		(IN)	0	1987	-632	25	438	25	-609	25
VITI LEVU 1916	MVS	Clarke 1880								
Viti Levu Island (Fiji Islands)		(CD)	0	1987	51	25	391	25	-36	25
VOIROL 1874	VOI	Clarke 1880								
Tunisia/Algeria		(CD)	0	1997	-73	**	-247	**	227	**
VOIROL 1960	VOR	Clarke 1880								
Algeria		(CD)	0	1993	-123	25	-206	25	219	25
WAKE-ENIWETOK 1960	ENW	Hough								
Marshall Islands		(HO)	0	1991	102	3	52	3	-38	3
WAKE ISLAND ASTRO 1952	WAK	International 1924								
Wake Atoll		(IN)	0	1991	276	25	-57	25	149	25
YACARE	YAC	International 1924								
Uruguay		(IN)	0	1987	-155	**	171	**	37	**
ZANDERIJ	ZAN	International 1924								

TABLE B.1. Ellipsoid center shift transformation parameters, local geodetic system to WGS 84.

Local Geodetic System		Reference Ellipsoid (ID Code)	Publication		Ellipsoid Center Shift (meters) *					
Name	Code		Cycle	Date	ΔX	$\sigma_{\Delta X}$	ΔY	$\sigma_{\Delta Y}$	ΔZ	$\sigma_{\Delta Z}$
Suriname		(IN)	0	1987	-265	5	120	5	-358	8

APPENDIX C:
COUNTRIES AND ASSOCIATED LOCAL GEODETIC SYSTEMS

10   GENERAL

10.1   Scope. This appendix consists of a list of countries and geographic regions with associated local geodetic systems.

20 APPLICABLE DOCUMENTS

The information in this section was extracted from NGA TR8350.2 (Third Edition).

30 GENERAL REQUIREMENTS

30.1   Application. Table C.1 provides a list of countries or geographic areas for quick reference to possible associated local geodetic systems (datums). Worldwide geodetic systems (e.g. WGS 72 and WGS 84), as well as other local geodetic systems, also may apply in the listed countries/areas. Country names and boundaries change over time. Do not use this list to program automatic selection of datums by geographic area or country name.

TABLE C. 1. Countries and associated local geodetic systems.

COUNTRY/REGION	LOCAL GEODETIC SYSTEM(S)
Afghanistan	HERAT NORTH
Alaska	NORTH AMERICAN 1927, 1983
Albania	S-42 (PULKOVO 1942)
Aleutian Islands	NORTH AMERICAN 1927, 1983
American Samoa Islands	AMERICAN SAMOA 1962
Algeria	NORTH SAHARA 1959
	VOIROL 1874, 1960
Antarctica Camp McMurdo Area	CAMP AREA ASTRO
Antigua Island	ANTIGUA ISLAND ASTRO 1943
	NORTH AMERICAN 1927
Argentina	CAMPO INCHAUSPE 1969
	SOUTH AMERICAN 1969
	SOUTH AMERICAN GEOCENTRIC REFERENCE SYSTEM (SIRGAS)
Ascension Island	ASCENSION ISLAND 1958

TABLE C. 1. Countries and associated local geodetic systems.

COUNTRY/REGION	LOCAL GEODETIC SYSTEM(S)
Australia	AUSTRALIAN GEODETIC 1966, 1984
Austria	EUROPEAN 1950, 1979
Azores	
Corvo Island	OBSERVATORIO METEOROLOGICO 1939
Faial Island	GRACIOSA BASE SW 1948
Flores Island	OBSERVATORIO METEOROLOGICO 1939
Graciosa Island	GRACIOSA BASE SW 1948
Pico Island	GRACIOSA BASE SW 1948
Santa Maria Island	SAO BRAZ
Sao Jorge Island	GRACIOSA BASE SW 1948
Sao Miguel Island	SAO BRAZ
Terceira Island	GRACIOSA BASE SW 1948
Bahama Islands	CAPE CANAVERAL
	NORTH AMERICAN 1927
Bahrain Island	AIN EL ABD 1970
Baltra, Galopagos Islands	SOUTH AMERICAN 1969
Bangladesh	INDIAN
Banka Island (Indonesia)	BUKIT RIMPAH
Barbados	NORTH AMERICAN 1927
Barbuda	NORTH AMERICAN 1927
Belgium	EUROPEAN 1950
Belize	NORTH AMERICAN 1927, 1983
Bermuda Islands	BERMUDA 1957
Bolivia	PROVISIONAL SOUTH AMERICAN 1956
	SOUTH AMERICAN 1969
	SOUTH AMERICAN GEOCENTRIC REFERENCE SYSTEM (SIRGAS)
Bosnia and Herzegovina	HERMANNSKOGEL
Botswana	ARC 1950
Brazil	CORREGO ALEGRE
	SOUTH AMERICAN 1969
	SOUTH AMERICAN GEOCENTRIC REFERENCE SYSTEM (SIRGAS)
Brunei	TIMBALAI 1948
Burkina Faso	ADINDAN
	POINT 58
Burundi	ARC 1950

TABLE C. 1. Countries and associated local geodetic systems.

COUNTRY/REGION	LOCAL GEODETIC SYSTEM(S)
Caicos Islands	NORTH AMERICAN 1927
Cameroon	ADINDAN
	MINNA
Canada	NORTH AMERICAN 1927, 1983
Canary Islands	PICO DE LAS NIEVES
Caroline Islands (Federated States of Micronesia)	KUSAIE ASTRO 1951
Cayman Brac Island	L.C. 5 ASTRO 1961
Channel Islands	EUROPEAN 1950
Chatam Island (New Zealand)	CHATHAM ISLAND ASTRO 1971
Chile	PROVISIONAL SOUTH AMERICAN 1956
	PROVISIONAL SOUTH CHILEAN 1963
	SOUTH AMERICAN 1969
	SOUTH AMERICAN GEOCENTRIC REFERENCE SYSTEM (SIRGAS)
Cocos Islands	ANNA 1 ASTRO 1965
Colombia	BOGOTA OBSERVATORY,
	PROVISIONAL SOUTH AMERICAN 1956
	SOUTH AMERICAN 1969
	SOUTH AMERICAN GEOCENTRIC REFERENCE SYSTEM (SIRGAS)
Con Son Islands (Vietnam)	INDIAN 1960
Congo	POINT NOIRE 1948
Costa Rica	NORTH AMERICAN 1927, 1983
Croatia	HERMANNSKOGEL
Cuba	NORTH AMERICAN 1927
Cyprus	EUROPEAN 1950
Czechoslovakia	S-42 (PULKOVO 1942)
(Before 1 January 1993)	S-JTSK
Deception Island, Antarctica	DECEPTION ISLAND 1969
Denmark	EUROPEAN 1950
Diego Garcia	ISTS 073 ASTRO 1969
Djibouti	AYBELLE LIGHTHOUSE
Dominican Republic	NORTH AMERICAN 1927
Easter Island	EASTER ISLAND 1969
East Falkland Island	SAPPER HILL 1943
East Malaysia (Sarawak & Sabah)	TIMBALAI 1948

TABLE C. 1. Countries and associated local geodetic systems.

COUNTRY/REGION	LOCAL GEODETIC SYSTEM(S)
Ecuador	PROVISIONAL SOUTH AMERICAN 1956 SOUTH AMERICAN 1969
Efate Island	BELLEVUE (IGN)
Egypt	EUROPEAN 1950 OLD EGYPTIAN 1907
El Salvador	NORTH AMERICAN 1927, 1983
England	EUROPEAN 1950 ORDNANCE SURVEY OF GREAT BRITAIN 1936
Erromango Island	BELLEVUE (IGN)
Eritrea (Ethiopia)	MASSAWA
Espirito Santo Island	SANTO (DOS) 1965
Estonia	COORDINATE SYSTEM 1937 OF ESTONIA
Ethiopia	ADINDAN
Federal Republic of Germany (before 1 January 1993)	EUROPEAN 1950
Finland	EUROPEAN 1950, 1979
France	EUROPEAN 1950
Gabon	M'PORALOKO
Ghana	LEIGON
Gibraltar	EUROPEAN 1950
Gizo Island (New Georgia Islands)	DOS 1968
Grand Cayman	NORTH AMERICAN 1927
Greece	EUROPEAN 1950

Greenland
   Hayes Peninsula
   South Greenland
	
	NORTH AMERICAN 1927
	QORNOQ

Guadalcanal Island	GUX I ASTRO
Guam	GUAM 1963
Guatemala	NORTH AMERICAN 1927, 1983
Guinea	DABOLA
Guinea-Bissau	BISSAU
Guyana	PROVISIONAL SOUTH AMERICAN 1956 SOUTH AMERICAN 1969
Honduras	NORTH AMERICAN 1927, 1983
Hong Kong	HONG KONG 1963
Hungary	S-42 (PULKOVO 1942)

TABLE C. 1. Countries and associated local geodetic systems.

COUNTRY/REGION	LOCAL GEODETIC SYSTEM(S)
Iceland	HJORSEY 1955
India	INDIAN
Indonesia	INDONESIAN 1974
Iran	EUROPEAN 1950
Iraq	EUROPEAN 1950
Ireland	EUROPEAN 1950
	IRELAND 1965
Isle of Man	ORDNANCE SURVEY OF GREAT BRITAIN 1936
Israel	EUROPEAN 1950
Italy	EUROPEAN 1950
Iwo Jima	ASTRO BEACON "E" 1945
Jamaica	NORTH AMERICAN 1927
Japan	TOKYO
Johnston Island	JOHNSTON ISLAND 1961
Jordan	EUROPEAN 1950
Kalimantan Island (Indonesia)	GUNUNG SEGARA
Kazakstan	S-42 (PULKOVO 1942)
Kenya	ARC 1960
Kerguelen Island	KERGUELEN ISLAND 1949
Korea (South)	TOKYO
	KOREAN GEODETIC DATUM 1995
Kuwait	EUROPEAN 1950
Latvia	S-42 (PULKOVO 1942)
Lebanon	EUROPEAN 1950
Leeward Islands	FORT THOMAS 1955
	ANTIGUA ISLAND ASTRO 1943
	MONTSERRAT ISLAND ASTRO 1958
Lesotho	ARC 1950
Liberia	LIBERIA 1964
Luxembourg	EUROPEAN 1950
Madagascar	TANANARIVE OBSERVATORY
Madeira Islands	PORTO SANTO 1936
Mahe Island	MAHE 1971
Malawi	ARC 1950
Malaysia Sarawak and Sabah	TIMBALAI 1948

TABLE C. 1. Countries and associated local geodetic systems.

COUNTRY/REGION	LOCAL GEODETIC SYSTEM(S)
Maldives, Republic of	GAN 1970
Mali	ADINDAN
Malta	EUROPEAN 1950
Marcus Island	ASTRONOMICAL STATION 1952
Marshall Islands	WAKE-ENIWETOK 1960
Mascarene Island	REUNION
Masirah Island (Oman)	NAHRWAN
Mexico	NORTH AMERICAN 1927, 1983
Midway Island	MIDWAY ASTRO 1961
Montserrat (Leeward Islands)	MONTSERRAT ISLAND ASTRO 1958
Morocco	MERCHICH
Namibia	SCHWARZECK
Nepal	INDIAN
Netherlands	EUROPEAN 1950, 1979
Nevis	FORT THOMAS 1955
New Zealand	GEODETIC DATUM 1949
Nicaragua	NORTH AMERICAN 1927, 1983
Niger	POINT 58
Nigeria	MINNA
Norway	EUROPEAN 1950, 1979
Okinawa	TOKYO
Oman	OMAN
Masirah Island	NAHRWAN
Pakistan	INDIAN
Panama (Canal Zone)	NORTH AMERICAN 1927, 1983
Paraguay	CHUA ASTRO
	SOUTH AMERICAN 1969
	SOUTH AMERICAN GEOCENTRIC REFERENCE SYSTEM (SIRGAS)
Peru	PROVISIONAL SOUTH AMERICAN 1956
	SOUTH AMERICAN 1969
	SOUTH AMERICAN GEOCENTRIC REFERENCE SYSTEM (SIRGAS)
Philippines	LUZON
Phoenix Islands	CANTON ASTRO 1966
Pitcairn Island	PITCAIRN ASTRO 1967
Poland	S-42 (PULKOVO 1942)

TABLE C. 1. Countries and associated local geodetic systems.

COUNTRY/REGION	LOCAL GEODETIC SYSTEM(S)
Porto Santo Island	PORTO SANTO 1936
Portugal	EUROPEAN 1950
Puerto Rico	PUERTO RICO
Qatar	QATAR NATIONAL
Romania	S-42 (PULKOVO 1942)
Russia	PULKOVO 1942
Salvage Islands	SELVAGEM GRANDE 1938
San Salvador Island	NORTH AMERICAN 1927
Sarawak and Sabah (East Malaysia)	TIMBALAI 1948
Sardinia	EUROPEAN 1950
	ROME 1940
Saudi Arabia	AIN AL ABD 1970
	EUROPEAN 1950
	NAHRWAN
Scotland	EUROPEAN 1950
	ORDNANCE SURVEY OF GREAT BRITAIN 1936
Sierra Leone	SIERRA LEONE 1960
Senegal	ADINDAN
Serbia	HERMANNSKOGEL
Shetland Islands	EUROPEAN 1950
	ORDNANCE SURVEY OF GREAT BRITAIN 1936
Sicily	EUROPEAN 1950
Singapore	KERTAU 1948
	SOUTH ASIA
Slovenia	HERMANNSKOGEL
Somalia	AFGOOYE
South Africa	CAPE
South America	SOUTH AMERICAN GEOCENTRIC REFERENCE SYSTEM (SIRGAS)
South Georgia Island	ISTS 061 ASTRO 1968
Spain	EUROPEAN 1950, 1979
Sri Lanka	KANDAWALA
St. Helena Island	ASTRO DOS 71/4
St. Kitts	FORT THOMAS 1955
Sudan	ADINDAN
Sumatra (Indonesia)	DJAKARTA (BATAVIA)
Suriname	ZANDERIJ

TABLE C. 1. Countries and associated local geodetic systems.

COUNTRY/REGION	LOCAL GEODETIC SYSTEM(S)
Swaziland	ARC 1950
Sweden	EUROPEAN 1950, 1979
Switzerland	EUROPEAN 1950, 1979
Syria	EUROPEAN 1950
Taiwan	HU-TZU-SHAN
Tanzania	ARC 1960
Tasmania	AUSTRALIAN GEODETIC 1966, 1984
Tern Island	ASTRO TERN ISLAND (FRIG) 1961
Thailand	INDIAN 1954, 1975
Trinidad and Tobago	NAPARIMA, BWI SOUTH AMERICAN 1969
Tristan da Cunha	TRISTAN ASTRO 1968
Tunisia	CARTHAGE EUROPEAN 1950 VOIROL 1874
Turks Islands	NORTH AMERICAN 1927
United Arab Emirates	NAHRWAN

United States of America
    Continental U.S. (CONUS)
    Alaska
    Aleutian Islands
    Florida
 	
    Hawaii (Hawaii, Kauai, Maui, Oahu)
	
	NORTH AMERICAN 1927, 1983
	NORTH AMERICAN 1927, 1983
	NORTH AMERICAN 1927, 1983
	CAPE CANAVERAL
	NORTH AMERICAN 1927, 1983
	NORTH AMERICAN 1983
	OLD HAWAIIAN

Uruguay	YACARE SOUTH AMERICAN GEOCENTRIC REFERENCE SYSTEM (SIRGAS)
Venezuela	PROVISIONAL SOUTH AMERICAN 1956 SOUTH AMERICAN 1969 SOUTH AMERICAN GEOCENTRIC REFERENCE SYSTEM (SIRGAS)

Vietnam
    Vietnam near 16oN
    Con Son Island
	
	INDIAN 1960
	INDIAN 1960

Virgin Islands	PUERTO RICO
Viti Levu Island (Fiji Islands)	VITI LEVU 1916
Wales	ORDNANCE SURVEY OF GREAT BRITAIN 1936

TABLE C. 1. Countries and associated local geodetic systems.

COUNTRY/REGION	LOCAL GEODETIC SYSTEM(S)
Wake Atoll	WAKE ISLAND ASTRO 1952
West Malaysia	KERTAU 1948
Yugoslavia (before 1990)	HERMANNSKOGEL
Zaire	ARC 1950
Zambia	ARC 1950
Zimbabwe	ARC 1950

APPENDIX D:
WGS 84 GEOID SEPARATION COMPUTATION

10. GENERAL.

10.1   Scope. This appendix provides the bilinear method for interpolating geoid separation and WGS 84 Geoid Separations on a 10° x 10° grid. These separations are expressed in meters.

20. APPLICABLE DOCUMENTS.

The geoid separation values in Table D.1 are from DMA TR8350.2 (1988) and may differ from the more recent Earth Gravitational Model 1996 (EGM96). Software, the gravity model coefficients, and the 15 minute grid of EGM96 geoid separation values, accurate to 1 meter or better, is available at http://cddisa.gsfc.nasa.gov/926/egm96/egm96.html.

30. GENERAL REQUIREMENTS.

30.1 The Bilinear Interpolation Method of Calculating WGS 84 Geoid Separation ($N_{WGS\ 84}$).

The bilinear interpolation method uses interpolation in two directions (ϕ,λ) to approximate the geoid separation $N_{WGS\ 84}$, from four known WGS 84 geoid separations given in a look-up table.

The bi-linear interpolation method has the form given in equation set D.1. Figure D.1 shows the relationship between the positions of the known $N_{WGS\ 84}$ values and the interpolation point.

$$N_p\ (\phi,\lambda) = a_0 + a_1X + a_2Y + a_3XY \qquad\qquad (D.1)$$

where:

$N_p(\phi,\lambda)$ = Geoid Separation (N) to be interpolated at Point $P(\phi,\lambda)$.

$a_0 = N_1$

$a_1 = N_2 - N_1$

$a_2 = N_4 - N_1$

$a_3 = N_1 + N_3 - N_2 - N_4$

$$X = \frac{(\lambda - \lambda_1)}{(\lambda_2 - \lambda_1)}$$

$$Y = \frac{(\phi - \phi_1)}{(\phi_2 - \phi_1)}$$

ϕ = Geodetic latitude of Point P

λ = Geodetic longitude of Point P

N_1, N_2, N_3, N_4 = Known geoid heights at grid points used in the interpolation process.
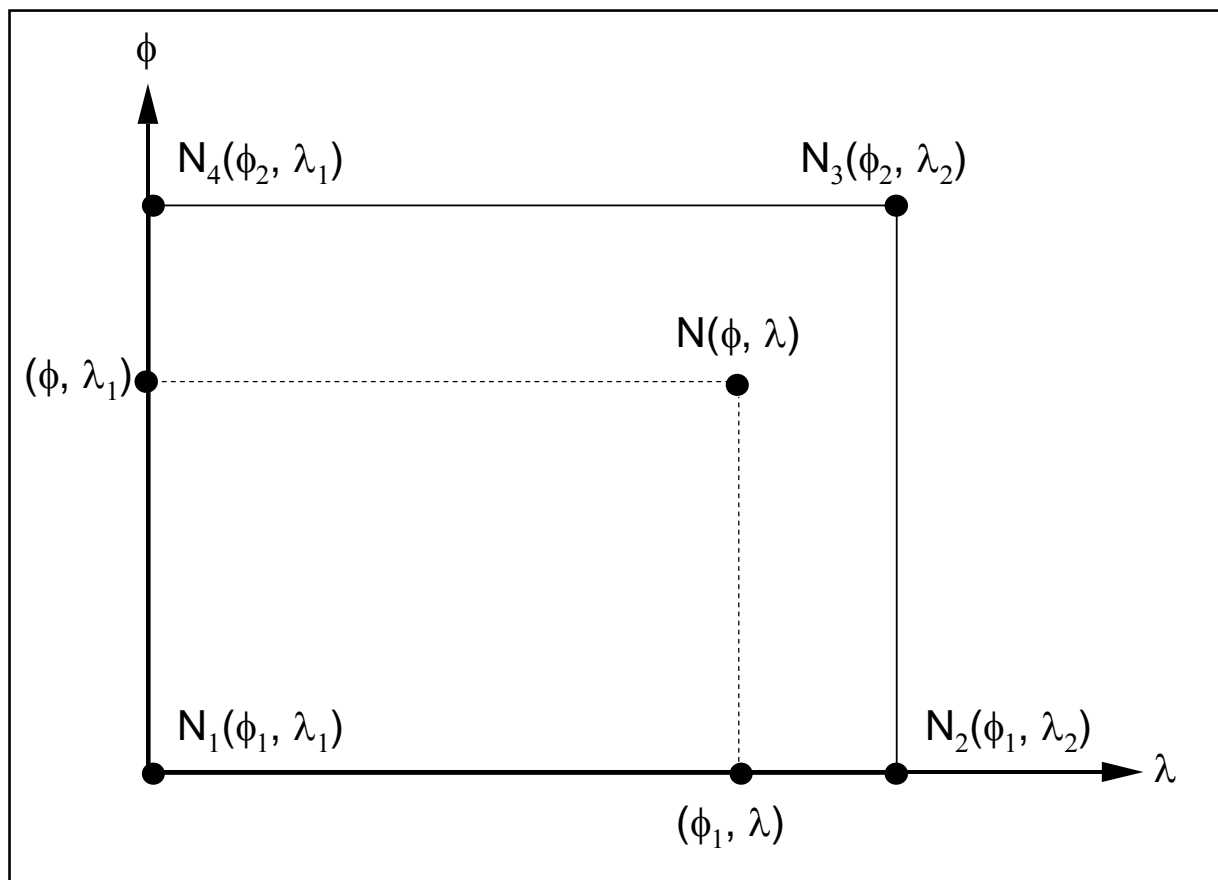
Figure D.1. Coordinate system associated with geoid seperation bi-linear interpolation method.

30.2 $\underline{N_{WGS\,84}}$ Tables. $N_{WGS\,84}$ values are tabulated on a 10° x 10° grid in Table D.1. The values in Table D.1 are suitable for use in moderate accuracy mapping and charting applications. EGM96 should be used for survey and high accuracy mapping and charting applications.

TABLE D.1. WGS 84 geoid separations (N = M = 180, 10° x 10° Grid).

Geoid Separation (meters)

Latitude (degrees)	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170
90	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13
80	33	34	28	23	17	13	9	4	4	1	-2	-2	0	2	3	2	1	1
70	51	43	29	20	12	5	-2	-10	-14	-12	-10	-14	-12	-6	-2	3	6	4
60	47	41	21	18	14	7	-3	-22	-29	-32	-32	-26	-15	-2	13	17	19	6
50	47	48	42	28	12	-10	-19	-33	-43	-42	-43	-29	-2	17	23	22	6	2
40	52	48	35	40	33	-9	-28	-39	-48	-59	-50	-28	3	23	37	18	-1	-11
30	36	28	29	17	12	-20	-15	-40	-33	-34	-34	-28	7	29	43	20	4	-6
20	31	26	15	6	1	-29	-44	-61	-67	-59	-36	-11	21	39	49	39	22	10
10	22	23	2	-3	-7	-36	-59	-90	-95	-63	-24	12	53	60	58	46	36	26
0	18	12	-13	-9	-28	-49	-62	-89	-102	-63	-9	33	58	73	74	63	50	32
-10	12	13	-2	-14	-25	-32	-38	-60	-75	-63	-26	0	35	52	68	76	64	52
-20	17	23	21	8	-9	-10	-11	-20	-40	-47	-45	-25	5	23	45	58	57	63
-30	22	27	34	29	14	15	15	7	-9	-25	-37	-39	-23	-14	15	33	34	45
-40	18	26	31	33	39	41	30	24	13	-2	-20	-32	-33	-27	-14	-2	5	20
-50	25	26	34	39	45	45	38	39	28	13	-1	-15	-22	-22	-18	-15	-14	-10
-60	16	19	25	30	35	35	33	30	27	10	-2	-14	-23	-30	-33	-29	-35	-43
-70	16	16	17	21	20	26	26	22	16	10	-1	-16	-29	-36	-46	-55	-54	-59
-80	-4	-1	1	4	4	6	5	4	2	-6	-15	-24	-33	-40	-48	-50	-53	-52
-90	-30	-30	-30	-30	-30	-30	-30	-30	-30	-30	-30	-30	-30	-30	-30	-30	-30	-30

Longitude (degrees east)

TABLE D.1. WGS 84 geoid separations (continued) (N = M = 180, 10° x 10° Grid)

Geoid Separation (meters)																	
90	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13
80	3	1	-2	-3	-3	-3	-1	3	1	5	9	11	19	27	31	34	33	34
70	2	2	1	-1	-3	-7	-14	-24	-27	-25	-19	3	24	37	47	60	61	58
60	2	9	17	10	13	1	-14	-30	-39	-46	-42	-21	6	29	49	65	60	57
50	-8	8	8	1	-11	-19	-16	-18	-22	-35	-40	-26	-12	24	45	63	62	59
40	-12	-10	-13	-20	-31	-34	-21	-16	-26	-34	-33	-35	-26	2	33	59	52	51
30	-7	-5	-8	-15	-28	-40	-42	-29	-22	-26	-32	-51	-40	-17	17	31	34	44
20	5	10	7	-7	-23	-39	-47	-34	-9	-10	-20	-45	-48	-32	-9	17	25	31
10	13	12	11	2	-11	-28	-38	-29	-10	3	1	-11	-41	-42	-16	3	17	33
0	22	16	17	13	1	-12	-23	-20	-14	-3	14	10	-15	-27	-18	3	12	20
-10	36	22	11	6	-1	-8	-10	-8	-11	-9	1	32	4	-18	-13	-9	4	14
-20	51	27	10	0	-9	-11	-5	-2	-3	-1	9	35	20	-5	-6	-5	0	13
-30	46	22	5	-2	-8	-13	-10	-7	-4	1	9	32	16	4	-8	4	12	15
-40	21	6	1	-7	-12	-12	-12	-10	-7	-1	8	23	15	-2	-6	6	21	24
-50	-15	-18	-18	-16	-17	-15	-10	-10	-8	-2	6	14	13	3	3	10	20	27
-60	-45	-43	-37	-32	-30	-26	-23	-22	-16	-10	-2	10	20	20	21	24	22	17
-70	-61	-60	-61	-55	-49	-44	-38	-31	-25	-16	-6	1	4	5	4	2	6	12
-80	-53	-54	-55	-52	-48	-42	-38	-38	-29	-26	-26	-24	-23	-21	-19	-16	-12	-8
-90	-30	-30	-30	-30	-30	-30	-30	-30	-30	-30	-30	-30	-30	-30	-30	-30	-30	-30
	180	190	200	210	220	230	240	250	260	270	280	290	300	310	320	330	340	350

Latitude (degrees)

Longitude (degrees east)

APPENDIX E:
SEVEN-PARAMETER GEOMETRIC DATUM SHIFTS

10. GENERAL

10.1 Scope. This appendix provides the data needed to perform seven-parameter geometric datum shifts between two local geodetic systems and WGS 84.

20. APPLICABLE DOCUMENTS.

This section is not applicable to this appendix.

30. GENERAL REQUIREMENTS

30.1 Application. The coverage area, ellipsoid applicable to the local geodetic system, ellipsoid center shifts (ΔX, ΔY, ΔZ), axes rotations (ε, ψ, ω), and scale factor change (ΔS) for use in seven-parameter geometric datum shifts between two local geodetic systems and WGS 84 are listed below.   The root-mean square differences between Doppler-derived WGS 84 geodetic coordinates and WGS 84 coordinates, computed from local system coordinates using the seven-parameter model, are provided as an indication of the datum shift consistency.

DATUM: EUROPEAN 1950 (EUR-M)

COVERAGE AREA: Western Europe

ELLIPSOID: International

$\Delta X = -102$ m $\Delta Y = -102$ m $\Delta Z = -129$ m

$\varepsilon = 0.413"$ $\psi = -0.184"$ $\omega = 0.385"$ $\Delta S = 2.4664 \times 10^{-6}$

RMS DIFFERENCES: WGS 84 surveyed, minus computed coordinates

$\Delta\phi = 2$ m $\Delta\lambda = 3$ m $\Delta H = 2$ m

DATUM: ORDNANCE SURVEY OF GREAT BRITAIN 1936

COVERAGE AREA: England, Isle of Man, Scotland, Shetland Islands, and Wales

ELLIPSOID: Airy

$\Delta X = 446$ m $\Delta Y = -99$ m $\Delta Z = 544$ m

$\varepsilon = -0.945"$ $\psi = -0.261"$ $\omega = -0.435"$ $\Delta S = -20.8927 \times 10^{-6}$

RMS DIFFERENCES: WGS 84 surveyed minus computed coordinates

$\Delta\phi = 2$ m $\quad$ $\Delta\lambda = 2$ m $\quad$ $\Delta H = 1$ m

APPENDIX F:
OLD HAWAIIAN DATUM WITH INTERNATIONAL ELLIPSOID

10.   GENERAL

10.1   Scope. This appendix provides the information needed to shift coordinates between the Clarke 1866 and International in the Old Hawaiian datum.

20.   APPLICABLE DOCUMENTS

DMA SGG MEMORANDOM FOR RECORD
SUBJECT: Old Hawaiian Datum with International Ellipsoid, 13 June 1990

Army Map Service GEODETIC MEMO No. 687

SUBJECT: Hawaiian Islands, UTM Coordinates and International Spheroid Geographic Positions. P.O.# 23832-002, 29 November 1950.

30.   GENERAL REQUIREMENTS

30.1   Background. Positions on the Old Hawaiian datum may be on either the Clarke 1866 ellipsoid or the International ellipsoid. Users of Old Hawaiian datum positions must determine which ellipsoid applies. Datum shift parameters from Old Hawaiian datum to WGS 84 have been determined only for the Clarke 1866 ellipsoid. Old Hawaiian datum coordinates on the international ellipsoid must be converted to the Clarke 1866 ellipsoid before being shifted to WGS 84. The conversion between International and Clarke 1866 ellipsoids has been defined in terms of UTM coordinates. Geodetic latitude and longitude must be converted to UTM before changing ellipsoids. Height conversions should not be attempted, as no conversion method is available.

30.2   Application. To convert Old Hawaiian datum UTM coordinates on the International ellipsoid (E_{INT}, N_{INT}) to UTM coordinates on the Clarke 1866 ellipsoid (E_{C66}, N_{C66}) use:

In UTM Zone 4

$$E_{C66} = E_{INT} - 3 \tag{F.1}$$

$$N_{C66} = N_{INT} - 169$$

In UTM Zone 5

$$E_{C66} = E_{INT} + 13 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\text{(F.2)}$$

$$N_{C66} = N_{INT} - 169$$

To convert Old Hawaiian datum UTM coordinates on the Clarke 1866 ellipsoid to the International ellipsoid use:

In UTM Zone 4

$$E_{INT} = E_{C66} + 3 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\text{(F.3)}$$

$$N_{INT} = N_{C66} + 169$$

In UTM Zone 5

$$E_{INT} = E_{C66} - 13 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\text{(F.4)}$$

$$N_{INT} = N_{C66} + 169$$

30.3   Example. Convert geodetic survey coordinates on the Old Hawaiian datum, International ellipsoid, to WGS 84 geodetic coordinates.

$$\phi_{INT} = 21°\ 19'\ 37.431"N$$

$$\lambda_{INT} = 157°\ 58'\ 25.610"\ W \qquad\qquad \text{(This position is on the island of Oahu.)}$$

a. Convert ϕ_{INT}, λ_{INT} to UTM:

Zone = 4

$$E_{INT} = 606,428$$

$$N_{INT} = 2,358,722$$

b. Convert E_{INT}, N_{INT} to the Clarke 1866 ellipsoid using equation set F.1, for zone 4.

$$E_{C66} = 606,425$$

$$N_{C66} = 2,358,553$$

c. Convert E_{C66}, N_{C66} to geodetic coordinates on the Clarke 1866 ellipsoid.

$$\phi_{C66} = 21°\ 19'\ 37.425"N$$

$\lambda_{C66} = 157° \ 58' \ 25.631''W$

d. $\Delta X, \Delta Y, \Delta Z$ shift parameters for datum identifier code OHA-D should be used for shifting geodetic survey coordinates on Oahu.

Shift $\phi_{C66}, \lambda_{C66}$ to WGS 84 using, for example, the three-step method.

$\phi_{WGS \ 84} = 21° \ 19' \ 26.069'' \ N$

$\lambda_{WGS \ 84} = 157° \ 58' \ 15.766'' \ W$

INDEX

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14. ABSTRACT

This document provides U.S. Army organizations and agencies with general guidance on selecting the appropriate methods for shifting between local geodetic datums and the World Geodetic System (WGS), and for converting Cartesian and map projection coordinates to and from geodetic coordinates. This guidance is provided to aid the Army community in selecting datum shift algorithms; developing, selecting, and maintaining software using these algorithms; and implementing this software to support operational units. Equations are furnished for map projections and datums commonly used within the Army. References are provided for other, less commonly encountered, map projections and datums.

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